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The economic order quantity taking into account additional information about the known quantile of the cumulative distribution function of the product's sales volume

Zhanna N. Zenkova
E-mail: zhanna.zenkova@mail.tsu.ru

Wilson Musoni
E-mail: wmusoni@uok.ac.rw

National Research Tomsk State University,
Address: 36, Lenin Street, Tomsk 634030, Russia

Abstract

In modern logistics and supply chain management, the task of inventory management is paramount. The total costs of the enterprise and consequently, its profit, directly depend on the accuracy of calculating the volumes and terms of orders. In this work, the problem of increasing the accuracy of calculating the economic order quantity for a product was solved by involving additional information about the known quantile of a given level of the distribution function of the volume of product's demand. The quantile information was used to recalculate the annual demand for the product, based on a modified estimator of the sales expectation for the period. The modified estimator is asymptotically unbiased, normal, and more accurate than the traditional sample mean in the sense of mean squared error. New formulas for calculating the economic order quantity and its confidence interval were presented and tested on real data on the monthly sales volumes of goods of a large retail store network over two years. It is shown that the classic way of mean calculation led to an underestimation of the volume of the economic order quantity, which in turn increased the risk of a shortage, and hence a drop in the quality of logistics services. The new calculation method also showed that the period between orders should be one day shorter. The work is practically significant; according to its results, recommendations are given to the enterprise.

Key words: economic order quantity; sample mean; additional information; quantile; modified mean estimator; modified confidence interval; assessment accuracy; mean squared error.

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Introduction

In the struggle for the attention of consumers, the quality of the service provided to them and an increase in the level of their satisfaction [1], modern enterprises are increasingly focused on improving the accuracy of forecasting future demand for a product, while reducing the risks of shortages and surpluses, trying to introduce pulling production systems throughout the supply chain and building all production processes based on the final need for the product [2]. Knowing quite accurately the volume of future needs, an enterprise can ensure the supply of raw materials, details and goods with minimal costs and minimal risks of disruption of the production process due to a shortage of goods or overstocking of warehouses, moreover, almost exactly on time [3, 4]. Note that in this case trade is also considered as production. Therefore, in modern logistics, in order to search for the best estimate of potential demand, all kinds of mathematical models of various levels of complexity [5–8] are used, including those that involve additional information to reduce the forecasting error and improve the quality of assessment [9–11] and product classification [6, 12].

This paper proposes a new, more accurate method for calculating the economic order size and its confidence intervals using additional information about the quantile of the sales volume cumulative distribution function (CDF). The source of additional information can be logisticians, marketers, analysts of the company, who have a fairly broad knowledge of the specifics of the enterprise and the specifics of organizing the supply and storage of goods. Restrictive parameters are also taken into account, for example, the maximum volume of the warehouse and the observed frequency of cases of its overflow. Usually such knowledge is ignored, although it can be used to advantage, since it has long been known that additional information helps to improve the quality of statistical procedures [6–13]. It is also worth noting that the use of information about the quantile has already been considered in a number of works [13–18], using the method of projecting the estimate of the distribution function into an a priori class.

In our case, such a modified estimate of the mean is used, for which it is not required to first construct the modified empirical cumulative distribution function (EDF) and calculate the estimate of the mean value of the indicator by substituting the EDF into the integral of the mathematical expectation, which somewhat simplifies the calculations.

As a result, it should be emphasized that in this study, additional information made it possible to significantly reduce the mean squared error in assessing the annual demand for materials, and, therefore, to increase the accuracy when calculating the economic order quantity.

1. Economic order quantity with additional information about the known quantile of the cumulative distribution function of the product's sales volume

The calculation of an economic order quantity (EOQ) is a classic logistics problem [3]. Within the framework of this task, such a size of the order X_o is determined, which leads to the minimization of the total costs of its placement and storage, and ultimately affects not only the company's profit, but also its market value [19]. The size EOQ directly affects many logistics processes of the company and the logistics chain as a whole [20], up to the choice of packaging [21] and the mode of transport for delivery [22]. Inaccuracies in EOQ calculations can lead to the so-called bull-whip effect [23, 24], which often leads to colossal losses, as was first identified by Procter & Gamble when selling diapers [25, 26].

The classic formula for EOQ, taking into account the losses associated with the freezing of working capital, is obtained by minimizing the function of the annual total cost of purchasing and placing an order by *X*:

$$TC(X) = \frac{M}{X} \cdot k + \frac{X}{2} \cdot P \cdot (l + z), \quad (1)$$

where *M* is the annual demand for a product (stock);

k – is fixed costs for placing one order;

P – is the average annual cost of a unit of the considered product (stock);

z – is the share of the price not received due to the freezing of working capital in the stock of goods during a year (as a minimum value, we can consider the current refinancing rate or the size of the minimum rate on deposit accounts of commercial banks);

l – is the rate of storage costs during a year, a share of the price *P*.

In fact, the first term in the cost function (1), equal to $\frac{M}{X} \cdot k$ is the cost of placing orders in the amount of *X* during a year, while the number of orders per year $r = \frac{M}{X}$. The second term $\frac{X}{2} \cdot P \cdot (l + z)$, is the average cost of storing the stock during a year. It is easy to prove that the minimum value of the function of annual total costs is achieved if

$$TC'(X) = -\frac{M}{X^2} \cdot k + \frac{1}{2} \cdot P \cdot (l + z) = 0, \quad (2)$$

so

$$X_o = \sqrt{\frac{2Mk}{P(l+z)}}. \quad (3)$$

Note that formula (3) can be applicable only for a sufficiently stable demand for a product [27], i.e., actually for goods from group X according to the classification by the method of XYZ analysis [3, 4, 6]. In other words, the formula is applicable for such goods for which the coefficient of variation (CV) of a number of demand (sales) values does not exceed 10%. In this case, CV is expressed by the formula

$$CV = \frac{\sqrt{S^2}}{\overline{X}} \cdot 100\%. \quad (4)$$

In the formula (4)

$$\overline{X} = \frac{1}{N} \sum_{i=1}^{N} X_i \quad (5)$$

represents the average level of demand (sales),

$$S^2 = \frac{1}{N} \sum_{i=1}^{N} \left(X_i - \overline{X} \right)^2 \quad (6)$$

is the sample variance [28], X_1, X_2, ..., X_N are sales (demand) volumes for a product during the year, *N* – the number of considered periods during the year.

Suppose that the business experts of the company know that for a certain period the overall level of demand for the product did not exceed a certain threshold level x_q in $q \cdot 100\%$ cases. In fact, assuming that the volume of demand (sales) *X* is a random variable (RV) with CDF $F(x) = P(X \leq x)$, then such information can be presented in the form

$$F(x_q) = q, \quad (7)$$

where x_q is the known quantile of the CDF of a known level *q*.

We use additional information about the CDF quantile to improve the quality of EOQ estimation (3). To do this, assume that $\{X_1, X_2, \ldots, X_N\}$ is a sample of size N with independent, equally distributed RVs with CDF $F(x)$. Then you can find a more accurate estimate of the annual demand for a product (stock) using a modified estimate of the average level of demand, taking into account additional information of the form (7), according to the formula:

$$M^q = m \cdot \bar{X}^q, \tag{8}$$

where m is the number of periods in a year (for example, $m = 12$, if the average monthly demand is calculated),

$$\bar{X}^q = \frac{1}{N(N-1)} \cdot \sum_{i=1}^{N} \sum_{j=1, i\neq j}^{N} X_i \cdot \left(1 - \frac{\left(I_{(X_i < x_q)} - q\right) \cdot \left(I_{(X_j < x_q)} - q\right)}{q(1-q)}\right) \tag{9}$$

is a modified estimate of the average demand for the period [12]. This estimate is asymptotically unbiased and normal. Its variance is determined by the formula [12]:

$$Var\left\{\bar{X}^q\right\} = \sigma^2 - E^2\left\{\frac{X \cdot \left(I_{\{X<x_q\}} - q\right)}{q(1-q)}\right\} + O\left(\frac{1}{N}\right), \tag{10}$$

where

$$\sigma^2 = N \cdot Var\left\{\bar{X}\right\} = Var\{X\}. \tag{11}$$

Moreover, it is easy to verify using formula (10) that

$$\sigma_q^2 = \lim_{x\to\infty} N \cdot Var\left\{\bar{X}^q\right\} = \sigma^2 - \left(\sqrt{\frac{1-q}{q}} \cdot \int_{-\infty}^{x_q} x dF(x) - \sqrt{\frac{q}{1-q}} \cdot \int_{x_q}^{+\infty} x dF(x)\right)^2. \tag{12}$$

Hence, it is obvious that $\sigma_q^2 \leq \sigma^2$, and thus due to asymptotic unbiasedness for sufficiently large volumes of observations, the use of additional information about the known quantile leads to a decrease in the mean squared error (MSE) normalized to N:

$$N \cdot \text{MSE}\left\{\bar{X}^q\right\} = N \cdot E\left(\bar{X}^q - a\right)^2 \leq \sigma^2 = N \cdot E\left(\bar{X} - a\right)^2, \tag{13}$$

and hence, to improve the accuracy of estimating the average volume of demand. Here $a = EX$.

Note that the estimate of the mean with allowance for the quantile [13] obtained using the projection method [9] has a similar asymptotic variance (12). The advantage of estimate (9) is that it does not require any preliminary actions, namely, constructing an estimate of the EDF and its projection into the a priori class with the subsequent substitution of the modified estimate of the EDF into the integral for the mathematical expectation.

Figure 1 shows a graph of dependence σ_q^2 from q for $F(x) = U_{(0,1)}(x)$ is uniform in (0,1) CDF, for which $\sigma^2 = 1/12$. *Figure 2* shows a similar graph for $F(x) = N_{(0,1)}(x)$ – standard normal CDF with zero mean and variance $\sigma^2 = 1$. *Figure 3* shows the corresponding graph for the exponential distribution $F(x) = 1 - e^{-x}$, $x \geq 0$, with parameter $\lambda = 1$, $\sigma^2 = 1/\lambda = 1$. It can be seen from the graphs that taking into account additional information about the quantile made it possible to significantly improve the quality of estimation for all three cases, for sufficiently large volumes of observations N.

Knowing the asymptotic normality allows one to obtain confidence intervals with a confidence level γ for the modified mean (here L is the lower bound, H is the upper bound):

$$L^q = \bar{X}^q - \frac{z_\gamma \cdot \sigma_q}{\sqrt{N}}; \tag{14}$$

$$H^q = \bar{X}^q + \frac{z_\gamma \cdot \sigma_q}{\sqrt{N}}, \tag{15}$$

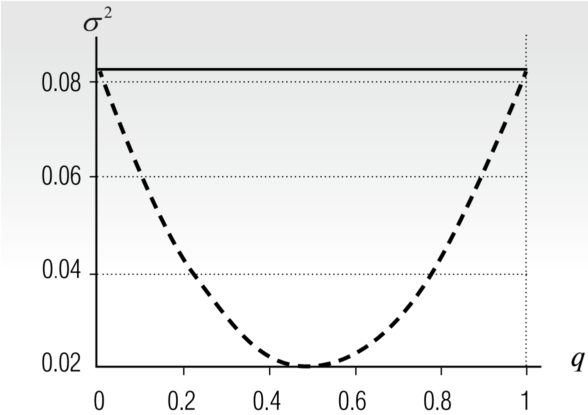

Fig. 1. The graph of σ_q^2 depending on q with $\sigma^2 = 1/12$, $F(x) = U_{(0,1)}(x)$

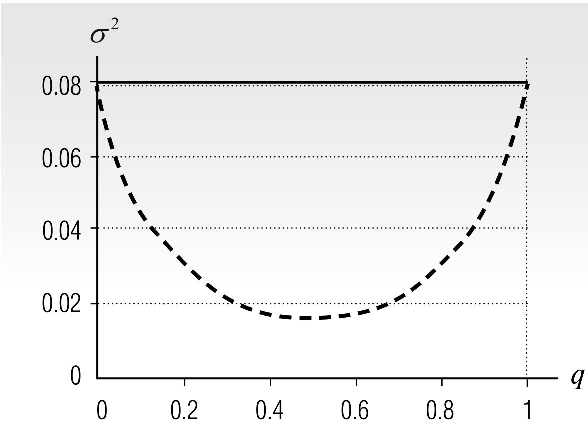

Fig. 2. The graph of σ_q^2 depending on q with $\sigma^2 = 1$, $F(x) = N_{(0,1)}(x)$

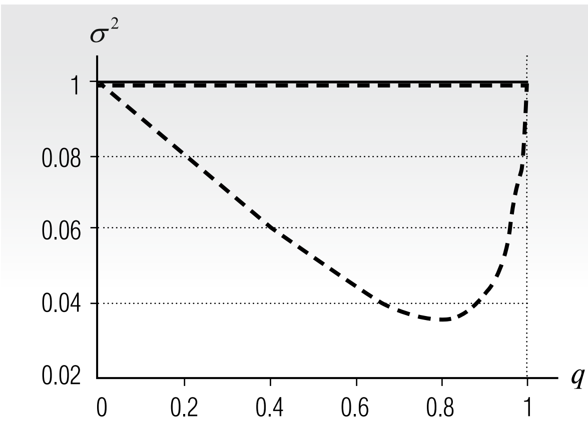

Fig. 3. The graph of σ_q^2 depending on q with $\sigma^2 = 1$, $F(x) = 1 - e^{-x}, x \ge 0$

where $\sigma_q = \sqrt{\sigma_q^2}$, σ_q^2 is calculated by the formula (12).

Thus, a more accurate modified formula for calculating the economic order quantity can be found using the formula:

$$X_o^q = \sqrt{\frac{2M^q \cdot k}{P(l+z)}}, \qquad (16)$$

where $M^q = m \cdot \bar{X}^q$, $\bar{X}^q$ is the average level of demand for a period, *m* is the number of periods in a year. Confidence interval with confidence level γ for modified economic order quantity are as follows:

$$L_{EOQ^q} = \sqrt{\frac{2mk}{P(l+z)}\left(\bar{X}^q - \frac{z_\gamma \cdot \sigma_q}{\sqrt{N}}\right)}; \qquad (17)$$

$$H_{EOQ^q} = \sqrt{\frac{2mk}{P(l+z)}\left(\bar{X}^q + \frac{z_\gamma \cdot \sigma_q}{\sqrt{N}}\right)}, \qquad (18)$$

where z_γ is the quantile of standard normal distribution with level $\frac{1+\gamma}{2}$.

Confidence level γ can be used as a measure for regulating the level of risk, it is established by the company's logisticians from the following considerations:

1. Since for highly profitable goods of group A (here AX), the deficit is critical and leads to direct losses [29], then it is recommended to take the upper bound H_{EOQ^q} as the basis for calculating the parameters of the inventory management system, the risk of a deficit is $R_D = \frac{1-\gamma}{2}$.

For example, if you need to provide a level of logistics service at which R_D is allowed at a level no more than 2% then $\gamma = 0.96$; for $R_D = 5\%$ $\gamma = 0.9$. However, it should be noted that the lower the risk R_D, the more γ, the wider the confidence interval, the greater the value H_{EOQ^q}, therefore, it is necessary to increase investments in stocks (this is a “payment” for a high level of service). Depending on the firm’s strategy H_{EOQ^q} is also used for calculating the parameters of the replenishment system for goods of group B (BX), but with a higher level R_D.

2. Lower bound L_{EOQ^q} is advisable to be used for low-profit products of group C (CX), for which it is extremely important to reduce the risk of surplus $R_S = \frac{1-\gamma}{2}$.

That is, if the company chooses a strategy of maximum customer satisfaction, then a higher level is chosen R_S, for example, $R_S = 10\%$, then you need to calculate the confidence limit L_{EOQ^q} with $\gamma = 0.8$. If the enterprise is aimed at reducing costs, then it is necessary to reduce R_S, for example, at $R_S = 1\%$ $\gamma = 0.98$, while an increase in γ is equivalent to a decrease in L_{EOQ^q}, and, therefore, a reduction in stocks. Depending on the firm’s strategy L_{EOQ^q} is also used for calculating the parameters of the replenishment system for goods of group B (BX), but with a lower level R_S.

Note that the equality of the levels of deficit risks R_D and surplus R_S is due to the symmetry of the standard normal distribution: the probability of “hitting” to the right of the upper boundary of the confidence interval is equal to “hitting” to the left of the lower boundary. Recall that the consideration here only of the groups AX, BX and CX, obtained according to the classification by the method of joint ABC–XYZ analysis [4, 6], is due to the fact that the calculation of the economic order quantity EOQ is permissible only for goods with stable demand from group X.

We also note that modern databases allow you to track the demand for a product (stock) with its detailing not only with accuracy of weeks or days, but also to hours. Thus, the number of periods can be considered large enough to use a more accurate formula (16).

2. Calculation of the economic order quantity based on real data

The modified calculation method proposed in the work was tested on real data on sales of a large retail chain. The monthly sales volumes for 2017–2018 were considered (a more detailed consideration here is inappropriate due to the loss of visualization of the application of the technique). *Table 1* and *Figure 4* show monthly sales values for two years and a graph of their dynamics. In order to preserve trade secrets, the name of the enterprise and the product is not specified, but it should be noted that the product is not perishable; its shelf life exceeds three years.

Table 1.

Sales of a product for 2017–2018, units / month

Year	2017	2018
January	9 081	7 953
February	7 578	6 267
March	7 578	7 700
April	8 044	7 747
May	8 490	7 780
June	7 587	7 110
July	7 031	7 392
August	7 560	7 328
September	8 258	7 776
October	7 609	8 586
November	7 501	8 302
December	7 898	8 999

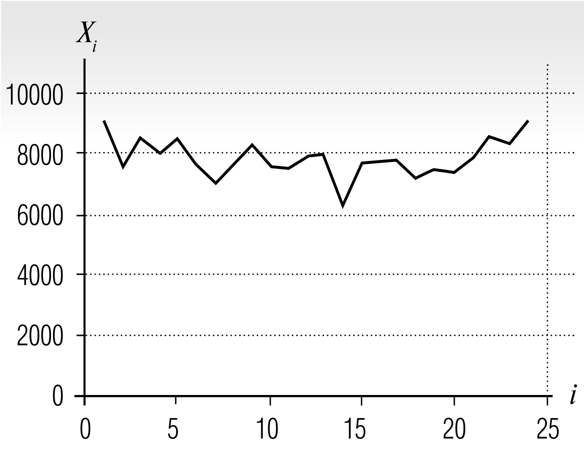

Fig. 4. Graph of the dynamics of sales for two years, units / month

Based on the given data, the average sales level (5) per month is $\overline{X} = 7\,835.75$ units, and the sample variance (6) $S^2 = 402\,546.98$ units in square, so the coefficient of variation (4) $CV = 8.10\%$, which allows us to conclude that the demand for this product is stable (the product belongs to group X). It is interesting to note that the data under consideration turned out to be normally distributed, which was confirmed by the Shapiro–Wilk test [30] with the calculated probability p-value = 0.691. *Figure 5* shows the EDF $F_N(x)$ graph (for convenience, the function is displayed as dots) and the corresponding normal CDF $N_{(7\,835{,}75;\ 402\,546{,}98)}(x)$.

The average annual demand for a product is $M = 94\ 029$ units. The company indicated the average price of the product for the period $P = 110$ rubles/unit, estimating storage costs at 50% of the price, i.e. $l = 0.5$ and let $z = 0.06$. The cost of placing one order is $k = 5\ 000$ rubles. As a result

$$X_o = \sqrt{\frac{2 \cdot 94\ 029 \cdot 5\,000}{110 \cdot 0.56}} = 3\ 906.97 \text{ units}.$$

As this product is indivisible, we will find the optimal whole X_o as one of the nearest integer arguments delivering the minimum of the total cost function:

$$TC(3\,906) = 240\ 669.623 > TC(3\,907) = \\ = 240\ 669.616 \text{ rubles/year}.$$

Eventually, $X_o^q = 3907$ units, therefore, the number of orders per year will be at least

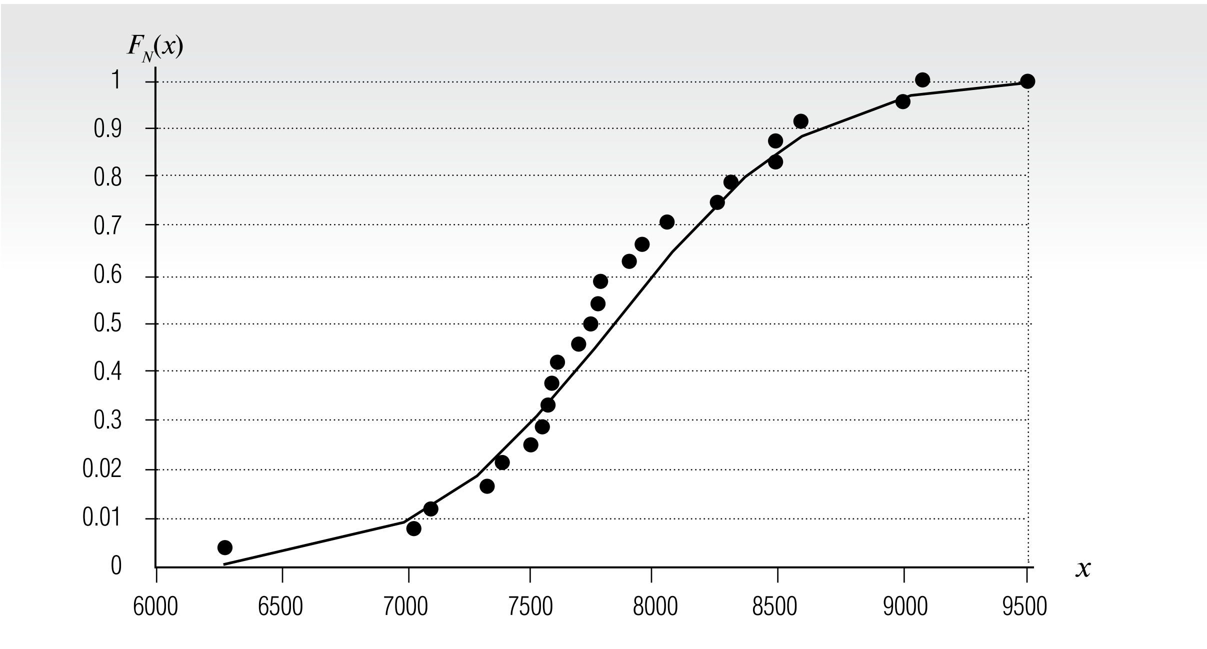

Fig. 5. Graphs of EDF $F_N(x)$ (dots) and CDF $N_{(7\,835{,}75;\ 402\,546{,}98)}(x)$ (solid line)

$r = \left\lceil \frac{94\ 029}{3907} \right\rceil = 24$ times with the frequency between orders $\left\lceil \frac{365}{24.07} \right\rceil = [15,\ 16] = 15$ days for category AX products and $\lceil 15,\ 16 \rceil = 16$ days for CX. For category BX, the choice of the number of days between deliveries (15 or 16) depends primarily on the strategic goals of the company.

When carrying out the calculations, the company provided additional information that for quite a long time (more than two years) in 95% of cases the monthly sales of this product did not exceed 9000 units per month, i.e. $F(x_q) = F(9000) = q = 0.95$. This allows a more accurate method of calculating the economic order size (16).

For the correct use of the modified statistics, the statistical hypothesis of the mutual independence of the data was tested by the series criterion [31], which confirmed the independence with the calculated probability *p*-value = 0.53161. As a result, it was found that $\bar{X}^q = 8\ 162.93$ units per month, $M^q = 12 \cdot 8\ 162.93 = 97\ 955.16$ units per year. Then $X_o^q = 3\ 987.71$ units per month, the minimum cost function is achieved at $X_o^q = 3\ 988$ units per month, the frequency of orders was $r^q = 24.56$, or at least 24 times a year. At the same time, the period between orders changed by 14.86 days. This means that if, according to the joint ABC–XYZ analysis, the product in question belongs to the AX group, then it must be ordered once every 14 days, for BX and CX – once every 15 days. *Figure 6* shows graphs of the dependencies of the total cost of restocking $TC(X)$ and $TC^q(X)$.

Note that the use of the modified method showed that the volume of the order was underestimated by 2.07% that could lead to a shortage of goods and to a loss of profit. As a result, the estimated total costs of the enterprise increased by 5 492.25 rubles / year, however, an improvement in the accuracy of estimating the annual demand for a product could potentially reduce logistics risks and compensate for losses due to higher profits, the increase of which is due to the higher quality of logistics services. We use the knowledge of the distribution of sales volumes of the initial data in order to obtain confidence intervals with the level of confidence γ for assessing the monthly average demand for a product (here the variance σ^2 is assumed to be known):

$$L = \bar{X} - \frac{z_\gamma \cdot \sigma}{\sqrt{N}}; \tag{19}$$

$$H = \bar{X} + \frac{z_\gamma \cdot \sigma}{\sqrt{N}}; \tag{20}$$

where z_γ is the $\frac{1+\gamma}{2}$; – level quantile of standard normal distribution; $\sigma = 634.47$ units per month.

To construct confidence intervals for the modified mean, using formula (12) at $F(x) = N_{(7\,835,75;402546.98)}(x)$, it was determined that $\sigma_q = \sqrt{\sigma_q^2} = 503.16$ units per month. The results are shown in *Table 2.*

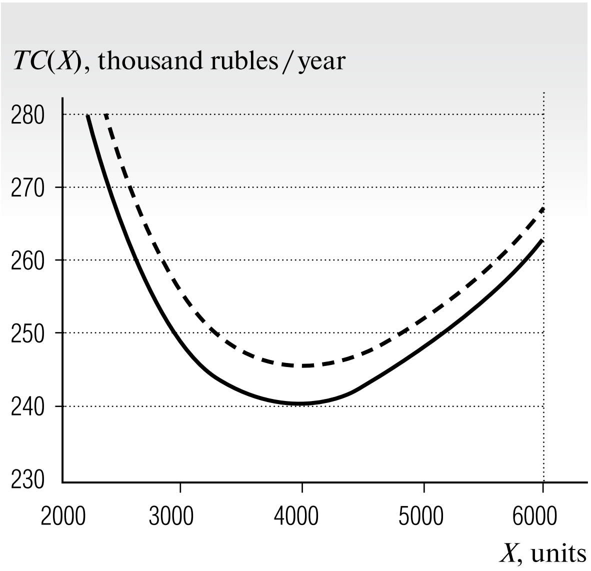

Fig. 6. Dependency plots $TC(X)$ (solid line) and $TC^q(X)$ (dotted line)

Table 2.

Confidence intervals for the average level of demand and economic order quantity with and without quantile information for different levels of confidence γ

γ	z_γ	L	H	L^q	H^q	L_{EOQ}	H_{EOQ}	L_{EOQ^q}	H_{EOQ^q}
0.98	2.326	7534	8138	7923	8402	3831	3982	3928	4046
0.95	1.960	7581	8090	7961	8365	3842	3970	3938	4037
0.9	1.645	7622	8049	7993	8332	3853	3960	3945	4029
0.8	1.282	7669	8002	8031	8295	3865	3949	3955	4020

For AX category products, it is recommended to use it as an economic order quantity H_{EOQ^q}. If the company seeks to meet the demand as much as possible, then large values of γ should be chosen, for example, 0.98. In this case, the volume of delivery will be 4 046 units, and the risk of a shortage R_D is no more than 1%. If the enterprise is aimed at reducing costs, then it needs to set higher values of R_D, for example, 5%. Then it has to choose $\gamma = 1 - 2 \cdot 0{,}05 = 0.9$, as a result, the economic order quantity must be 4 029 units. If our product belongs to the CX group, then it is advisable to minimize the permissible risk of surplus R_S, setting it at the level $\frac{1-\gamma}{2}$. Then, for example, at a 10% risk level $\gamma = 1 - 2 \cdot 0.1 = 0.8$ and the volume will be added to the order $L_{EOQ^q} = 3\ 955$ units. At the same time, it is expected that in two cases out of ten the demand for this product will be only partially satisfied, since it will be completely sold out and there will not be enough for all buyers.

It should also be noted that the confidence intervals with additional information are significantly narrower (by almost 22%) than the intervals without the quantile. This is due to the fact that the modified mean has an asymptotic variance $\sigma_q^2 \le \sigma^2$, so at the same level of deficit risks R_D and surplus R_S there can be found more accurate confidence limits – values of the economic order quantity for goods from different groups AX, BX or CX.

Conclusion

In this paper, we propose a new method for calculating the economic order quantity, taking into account additional information about the known quantile of a given level of the cumulative distribution function of sales (demand) volumes, as well as confidence intervals for it. We show that for a sufficiently large number of observations, the new method gives a more accurate value of the economic order quantity and narrower confidence intervals since it is based on an asymptotically unbiased estimate of the average level of demand for a product with a smaller mean squared error. The new calculation method was tested on real data on monthly sales of a large retail chain for two years. The company was given recommendations on the choice of parameters for the inventory management system of the product in question. ■

References

1. Johnson M.D., Gustafsson A. (2020) *Improving customer satisfaction, loyalty, and profit: An integrated measurement and management system*. Jossey-Bass.
2. Schönsleben P. (2016) *Integral logistics management: Operations and supply chain management within and across companies.* Boca Raton: CRC Press. DOI: 10.4324/9781315368320.
3. Silver E. A., Pyke D. F., Thomas D.J. (2017) *Inventory and production management in supply chains.* Boca Raton: CRC Press. DOI: 10.1201/9781315374406.
4. Zrilic A. (2015) *Six steps inventory optimization.* Lulu.
5. Tipi N. (2020) *Supply chain analytics and modelling: Quantitative tools and applications.* Kogan Page.
6. Zenkova Zh.N., Kabanova T.V. (2018) The ABC-XYZ analysis modified for data with outliers. Proceedings of the *4th IEEE International Conference on Logistics Operations Management (GOL' 2018), Le Havre, France, 10–12 April 2018*, pp. 63–68.
7. Gurov N.V., Zenkova Zh.N. (2016) Robust estimation of means in the analysis of the company's current assets turnover. Proceedings of the *V International Youth Scientific and Practical Conference on Mathematical and Computer Modeling in Economics, Insurance and Risk Management, Saratov, Russia, 9–12 November 2016*, pp. 47–51 (in Russian).
8. Zenkova Zh.N., Krakovetskaya I.V. (2013) Incomplete data modeling in logistics and marketing. Proceedings of the *International Scientific and Practical Conference on Logistics Systems in the Global Economy, Krasnoyarsk, Russia, 14–15 March 2013*, part 1, pp. 98–105 (in Russian).
9. Dmitriev Yu.G., Ustinov Yu.K. (1988) *Statistical estimation of probability distributions using additional information*. Tomsk: TSU (in Russian).
10. Dmitriev Yu.G., Koshkin G.M. (2018) Nonparametric estimators of probability characteristics using unbiased prior conditions. *Statistical Papers*, vol. 59, no 4, pp. 1559–1575. DOI: 10.1007/s00362-018-1044-7.
11. Dmitriev Yu.G., Tarasenko P.F. (1992) The use of a priori information in the statistical processing of experimental data. *Russian Physics Journal*, vol. 35, no 9, pp. 888–893. DOI: 10.1007/BF00560063.
12. Dmitriev Yu., Zenkova Zh., Musoni W. (2019) Statistical estimation with a known quantile and its application in a modified ABC-XYZ analysis. Book of abstracts of the *8th International Conference on Risk Analysis and Design of Experiments, Vienna, Austria, 23–27 April 2019*, pp. 143–144.
13. Zenkova Zh.N., Krainova E.A. (2017) Estimating the net premium using additional information about a quantile of the cumulative distribution function. *Business Informatics*, no 4, pp. 55–63. DOI: 10.17323/1998-0663.2017.4.55.63.
14. Makeeva O.B., Zenkova Zh.N. (2015) Calculation of turnover indicators based on interval-censored data, taking into account quantile knowledge. *Educatio*, no 8 (15), part 3, pp. 72–76 (in Russian).
15. Zenkova Zh.N., Makeeva O.B. (2015) Using quantile information in the analysis of working capital turnover. Proceedings of the *III All-Russian Youth Conference on Software and Programs for Information, Technical and Economic Systems, Tomsk, Russia, 22–23 May 2015*, pp. 82–87 (in Russian).
16. Zhurko E.S., Zenkova Zh.N. (2015) PSM pricing method for censored data with quantile consideration. *International Union of scientists "Science. Technologies. Production"*, no 9 (13), pp. 13–16 (in Russian).
17. Zhurko E.S., Zenkova Zh.N. (2016) Modification of the PSM pricing method taking into account the quantile of a given level. Proceedings of the *International Scientific and Practical Conference on Information Technologies in Siberia (ITSIBERIA–2016), Kemerovo, Russia, 10* November *2016*, pp. 134–136 (in Russian).
18. Kabanova S.A., Zenkova Zh.N., Danchenko M.A. (2018) Regional risks of artificial forestation in the steppe zone of Kazakhstan (case study of the green belt of Astana). *IOP Conference Series: Earth and Environmental Science*, vol. 211, no 1, article 012055, pp. 1–9. DOI: 10.1088/1755-1315/211/1/012055.
19. Serrano A., Kraiselburd S. (2007) Economic order quantity and the value of the firm. *SSRN Electronic Journal*, July, pp. 1–34. DOI: 10.2139/ssrn.1027547.

20. Mandeep M., Shah N.H. (2016) *Optimal inventory control and management techniques.* IGI Global. DOI: 10.4018/978-1-4666-9888-8.

21. McDonald C.M. (2016) Integrating packaging and supply chain decisions: Selection of economic handling unit quantities. *International Journal of Production Economics*, vol. 180, pp. 208–221. DOI: 10.1016/j.ijpe.2016.08.003.

22. Wasiak M. (2016) Vehicle selection model with respect to economic order quantity. *Archives of Transport*, vol. 40, no 4, pp. 77–85. DOI: 10.5604/08669546.1225471.

23. Durlinger P. (2020) *The bullwhip effect.* Durlinger Essential.

24. Snyder L.V., Shen Z.-J. M. (2019) *Fundamentals of supply chain theory*. Wiley.

25. Chopra S., Meindl P. (2007) Supply chain management. Strategy, planning & operation. *Das summa summarum des management* (C. Boersch, R. Elschen, eds.). Gabler, pp. 266–275.

26. Lee H.L., Padmanabhan V., Whang S. (1997) The bullwhip effect in supply chains. *MIT Sloan: Management Review*, vol. 38, no 3. Available at: https://sloanreview.mit.edu/article/the-bullwhip-effect-in-supply-chains/ (accessed 01 March 2020).

27. Wang C., Tang W., Zhao R. (2010) Analysis of economic order quantity under fuzzy environments. *Transactions of Tianjin University*, no 16, pp. 229–234.

28. Mittelhammer R.C. (2013) *Mathematical statistics for economics and business.* Springer Science & Business Media.

29. Mike P.C., Malburg T.W. (1981) Overcoming economic order quantity limitations. *Hospital Materiel Management Quarterly*, vol. 2, no 3, pp. 65–69.

30. Shapiro S.S., Wilk M.B. (1965) An analysis of variance test for normality (complete samples). *Biometrika*, vol. 52, no 3–4, pp. 591–611.

31. Ayvazyan S.A., Enyukov I.S., Meshalkin L.D. (1983) *Applied statistics: Fundamentals of modeling and primary data processing.* Moscow: Finance and Statistics (in Russian).

About the authors

Zhanna N. Zenkova

Cand. Sci. (Phys.-Math.), Associate Professor, MBA;

Associate Professor, Department of Systems Analysis and Mathematical Modeling,
Institute of Applied Mathematics and Computer Science,
National Research Tomsk State University,
36, Lenin Street, Tomsk 634030, Russia

E-mail: zhanna.zenkova@mail.tsu.ru

ORCID: 0000-0002-3776-9935

Wilson Musoni

Doctoral Student, Department of Systems Analysis and Mathematical Modeling,
Institute of Applied Mathematics and Computer Science,
National Research Tomsk State University,
36, Lenin Street, Tomsk 634030, Russia

E-mail: wmusoni@uok.ac.rw