Two Approaches to the Concept of Knowledge Application: Transfer and Modeling.

Overview and Criticism

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Abstract. The patterns of knowledge application in new situations are explored from the perspectives of modeling and transfer. We provide an overview of studies to compare these two conceptions and get a comprehensive idea of which psychological processes are involved in knowledge application, what will change in research and teaching practices if the conceptual frameworks change, and how these conceptions can contribute to each other. We show that analyzing the problem structure and comparing problem models in different representational systems are the key prerequisites for a successful knowledge application in both conceptions. Based on the data obtained, we draw conclusions about approaches to education promoting effective knowledge application and about training problem assessment criteria.

Keywords: learning, knowledge application, modeling, transfer, mathematical word problems, deep and surface structure, generalization, metacognitive skills.

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So far as education is concerned, whether school or professional, individual or mass, formal or informal, for children or adults, it is always implied that acquired knowledge and skills will be applied under conditions other than the learning environment. Otherwise speaking, the principal outcome of education should be the application of knowledge in new, unfamiliar situations. With a view to summarizing what is already known about the ability to use acquired knowledge beyond the educational context, we will consider the studies on modeling and transfer, the two psychological constructs immediately associated with the idea of knowledge application.

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The modeling conception has been historically focused on applying formal, "school" mathematical knowledge to informal, "real-life" (as opposed to "school") situations [Blum, Ferri, 2009; Frejd, 2013]. It was developed to solve a rather specific applied issue: school mathematical problems had nothing to do with the reality, leaving students unable to recognize the utility of mathematical construction in everyday life. Why are word problems singled out from the whole available wealth of school mathematical education in the modeling conception? The answer is: this is the only component of school mathematics designed to connect formal mathematics with real-life applications. Originally, word problems were aimed at teaching students to apply mathematical skills in real-life situations: in trade, tourism, construction, agriculture, military, etc. [Yushkevich, 1970]. As the corpus of purely mathematical knowledge developed, the connection toreal life problems was fading away and became absolutely delusive in present-day school mathematics, as is often observed (see, for instance, Arnold [1998]). Word problems remain the only syllabus component that preserves the connection of mathematical concepts with real-life needs, at least nominally. However, many studies show that such a connection is of a highly doubtful quality [Boaler, 1993; Verschaffel, Corte, 1993].

Nowadays, the approach to the application of mathematical knowledge as modeling may be considered commonly widespread. In trying to connect mathematics with real life, many countries have initiated relevant changes in their mathematical education programs [Freudenthal, 1973; 1991; Krauss et al., 2008; National Council of Teachers of Mathematics, 2006; YZZ, 2003]. Moreover, the modeling conception formed the basis for the PISA (Program for International Student Assessment) mathematics literacy test [OECD, 2013]. Yet, as we will see below, the growing popularity of this conception does not take into account its being restricted to the mathematical language, which makes it impossible to expand the results of modeling research to other subject areas. Additionally, the available empirical results still provide no clear understanding of the psychological processes subject to modeling or of the efficiency of the proposed approaches to education.

Research on transfer—another construct directly related to knowledge application—was pioneered by Edward Thorndike [Thorndike, 1924; Thorndike, Woodworth, 1901]. He gave perceptual tasks to test subjects and assessed how training to solve one type of problems improved solving other types. Since the first third of the last century, when Thorndike was working, the conceptual framework of transfer has expanded greatly in terms of both transfer object (procedural or representative skills, problem-solving approach, etc.) and transfer situations (transfer from academic context to everyday life, deferred and immediate transfer, etc.). Nevertheless, the key characteristic of understanding "application" through transfer remains the same: constructing an analogy between the learnt and the new and drawing inferences from this analogy to solve the new problem.

The transfer research empirical database is huge, but the information it provides has no direct relation to schooling. Yet, the research results can be justifiably extended to school knowledge since the transfer conception presents the transfer object and application context as universal phenomena. Another issue is that, unlike modeling, transfer is often described as a one-time action, which makes it barely teachable. Here, we should construct links between the well-described transfer mechanism and the teaching strategies.

The modeling and transfer conceptions provide different explanations of how acquired knowledge is applied and which factors determine its efficiency. We will perform a detailed comparison of these two approaches to identify the changes in research and teaching practices in different conceptual frameworks of knowledge application and to find out how the two can contribute to each other. The comparison is structured: for each of the conceptions, we analyzed the object, the context (parameters of the application situation), the process and the mechanism of application, as well as conducive learning, i. e. the proposed teaching methods that will facilitate further application of knowledge.

1. Modeling conception

1.1. Object

The modeling conception is based on the assumption that many processes and relations in physical, social, economic, personal and other spheres of life can be described with mathematical language, which allows for representing and solving a lot of problems at the abstract level. Hence, mathematical language is the object of application, i. e. what should be mastered in the learning context and then transferred to real life. The mathematical language represents a symbolic system used to describe mathematical objects and concepts, including, for example, numbers, function and conceptual symbols. The system contains both individual icons (+ or –) and complex graphic symbols. The language of mathematics should be mastered to describe and formalize what is going on in various areas, including those that are non-mathematical, like physics, economics or everyday life.

1.2. Process and mechanism

In the modeling conception, the application of the mathematical language comes down to performing subsequent actions to build an adequate mathematical model of a real-life situation: identifying the key elements of the problem and the links between them (problem structuring or situation model construction); encoding the situation model elements in mathematical terms (mathematical model construction); performing mathematical calculations and interpreting the solution in terms of the original "real-life" situation. Let us take, for instance, the following problem: *"Mrs. Stone lives in Trier, 20 km away from the border of Luxembourg. To fill up her VW Golf she drives to Luxembourg where immediatelyover the border there is a petrol station. There you have to pay €1.10 for one liter of petrol whereas in Trier you have to pay €1.35. Let us assume that the fuel consumption rate is 12 liters per 100 km. Is it worthwhile for Mrs. Stone to drive to Luxembourg? Give rea-*

sons for your answer." (citation from [Blum, Ferri, 2009]). In order to solve this problem, we should first of all make sense of the described situation, its conditions and requirements. It is clear that buying petrol at a cheaper station may be worthwhile or not depending on the difference in price and fuel consumption. Next, as we build a situation model, only the key links in the problems are left, for example: *if the cost of fuel (in euro) consumed to drive to the farther petrol station exceeds the saved money (the difference in petrol prices in euro), then driving to Luxembourg is not worthwhile*. At the stage of mathematization, the situation model is transformed into a mathematical model. As soon as the mathematical model has been constructed, using an inequality in this case, a calculation is performed. Next, the mathematical results should be interpreted back in the real world, ending up in a recommendation for Mrs. Stone what to do. To validate these results, the problem solver goes round the loop a second time to take into account any factors that may have been omitted.

Several scenarios of the modeling process have been suggested by now, yet all of them follow the same logic of three main steps: structuring, mathematizing and interpreting. In some cases, the first step is divided into understanding, simplifying and structuring. (For an overview of foreign studies, see, for instance, Borromeo Ferri [2006]; for an extensive description of modeling, especially at the stage of approaching the problem, by Russian researchers, see Galperin [1958]; Talyzina [2011], and Salmina [1988]; in some cases, a special focus is put on the stage of mathematical model transformation [Salmina, 1988]).

1.3. Context and application performance

Although the abstract language of mathematics is universally applicable, the modeling conception restricts application to the everyday context only, the so-called real life, rarely involving any other subject areas. In addition, the context is virtually reduced to the text of the problem, because modeling is studied and assessed using exclusively mathematical word problems as representative of real situations[1]. If the context, i.e. the area of knowledge application, comes down to the text of the problem, all the investigated factors affecting modeling performance should be inevitably rooted in this text.

The basic role of the text in problem solving is determined by its complementary status relative to the formal mathematical component of the problem. As a result, solving performance will be affected by (i) the formal mathematical aspect, mathematical difficulty of a problem, and solver's mathematical skills; and (ii) the linguistic component and reading comprehension skills. It turns out, however, that integration of the text and mathematical components produces the derivative third

[1] The important issue of prerequisites for representing a real-life situation is beyond the scope of this overview, primarily because we do not raise it.

component. The latter provides the unique difficulty of a problem that cannot be explained by either mathematical or linguistic components [Daroczy et al., 2015]. This derivative component determines the efficiency of performing two specific actions—constructing a situation model and encoding it in the mathematical language—which form the backbone of modeling. Formal mathematical difficulty lies beyond the scope of this article, so we will dwell on the factors related to reading comprehension and the modeling process.

Let us begin with the details in the problem statement. It turns out that details have no decisive impact on the correctness of problem solving, the effects depending on their relation to the text composition and the problem structure. Only details providing a clearer understanding of the problem structure facilitate solving. Meanwhile, solving does not become easier due to details that help imagine the relevant real-life situation better but do not clarify the problem structure [Davis-Dorsey, Ross, Morrison, 1991; Lepik, 1990; Vicente, Orrantia, Verschaffel, 2007].

The semantic properties of a text act as an independent factor. We demonstrate that semantics has an unconscious influence on mathematical model construction. For example, the use of functionally related items (boxes—oranges) activates the division model, while categorically related items (oranges—lemons) activate the addition model [Martin, Bassok, 2005].

Some turns of phrase in word problems give the solver translation cues that act as a trigger, translating the problem text automatically to a mathematical operation that is generally associated with specific words. For example, "times" → multiply, "together" → add [LeBlanc, Weber-Russell, 1996]. Obviously, such associations are justified by the experience of solving similar tasks, but the same cues may be misleading and complicating if they are inconsistent with the structure. There is a classic example of the misleading cue "twice as many" in the following problem: *"There are twice as many students as professors in a university"*, which activates the wrong mathematical model: $S \times 2 = P$, where S is the number of students and P is the number of professors.

In addition, not only individual phrases but the whole problem wording pattern may be a trigger activating a specific solving model. Again, it happens because a specific type of text pattern, or the "plot" of a problem, is normally associated with a specific mathematical model due to the accumulated experience in problem solving. We demonstrate that all word problems in school mathematics may be reduced to a set of standard "plots": distance-rate-time problems, river-rate problems, problems involving working together, and others. If the solver qualifies the text as a specific pattern, they may omit the modelingprocess and proceed to the final math model associated with this pattern [Blessing, Ross, 1996; Mayer, 1981].

The influence of problem-related illustrations on solving performance depends on whether they contain useful information or not.

Illustrations that do not carry unique information have no impact on the correctness of solving but increase the time spent to solve the problem. Illustrations featuring unique information required for solving, which are to be integrated with the text, complicate the task for the solver [Berends, van Lieshout, 2009]. Therefore, the distribution of information among different parts of the problem (e. g. partly in text, partly in drawing and partly in diagram) will complicate the solving process, short-term memory being busy bringing scattered information together.

Expectedly, irrelevant information purposefully included into a problem increases difficulty, but not only because it uses short-term memory resources. A qualitative analysis of problem-solving logs showed that students may be misguided by distracting information, e. g. getting them to think that they need to use all the numbers in the text or find another number if they are only given one [Muth, 1992].

The scope of research on this conception features very few empirical studies on specific modeling actions, and supporters regard it as a major hindrance [Borromeo Ferri, 2006]. The first stages of modeling, when the solver has to understand the conditions and construct a situation model of a problem, are believed to be the most challenging [De Corte, Verschaffel, Greer, 2000; Gürel, Gürses, Habibullin, 1995]. In particular, their difficulty is explained by the fact that the solver needs to be able to create short and precise mental representations, including visual ones, when constructing a situation model [Abdullah, Halim, Zakaria, 2014; Novick, Hmelo, 1994; Wertheimer, 1982; Zahner, Corter, 2010]. The solver is sometimes unable to choose or construct an effective representation of a problem. A number of studies reveal that representations may be incomplete or distorted, if indeed there are any at all, which affects the correctness of solving immediately [Tyumeneva, 2015; McGuinness, 1986; Novick, 1990; Wertheimer, 1982].

1.4. Conducive learning

Approaching the modeling process as part of a broader culture of the "conscious" teaching of mathematics, researchers attribute modeling performance to a successful realization of a whole package of teaching procedures, from teacher training to problem formulation. This package is usually described in such general terms that we can only use the same general terms to discuss the factors promoting the development of modeling skills. The whole package, sometimes referred to as *modeling discourse* [Niss, Blum, Galbraith, 2007], is designed to make both teachers and students understand the importance of modeling, to create a conducive learning environment to keep students involved, etc. [Blum, Ferri, 2009]. Due to the small amount of empirical research on the efficiency of the recommended practices and to the very general nature of such recommendations, it appears impossible to single out any specific factors contributing to the development of modeling skills. Such impossibility is sometimes considered to be fundamental [Ibid.]. This is why we can only name some specific features of syllabus organization and the activities actually used when teaching modeling skills and "realistic mathematics". Such activities include: providing students with a possibility to search for and establish the links between different mathematical areas as well as between mathematics and the world around on their own; treating modeling as part of the syllabus (similar to strategies, modeling skills are to be taught intentionally); teaching metacognitive skills (planning, breaking a problem into subproblems); encouraging various solution methods; minimizing teacher interference to allow for maximum independence of students in problem solving; providing metacognitive assistance (e. g. *"Imagine this situation", "What is your goal?", "Is your result consistent with this situation?*) [Reusser, 1996]. Many of these instructional techniques are in line with the ideas of effective learning developed within other pedagogical approaches, primarily the so-called constructivist-based pedagogy [Noddings, 1990; OECD, 2009]. In this context, the learning principles put forward by the modeling conception show weak inherent correlation with the conception itself, rather being represented as a modern philosophy of learning.

The inculcating approach to modeling proposed within the cultural and historical approach [Galperin, 1958; Talyzina, 2011, Fridman, 1977, Shevkin, 2005] is focused on the development of modeling skills as such. The studies place an emphasis on developing the learning program implementation techniques as mass-oriented. Just like their Western colleagues, the creators of the inculcating approach to modeling did not pay enough attention to assessing the efficiency of inculcating programs or their specific teaching techniques. Hence, although the established learning system is pretty consistent with the theoretical grounds of the cultural and historical approach, there is still little proof of its efficiency.

2. Transfer

2.1. Object

Modern research on transfer investigates a very wide array of skills as the object of transfer (for a comprehensive overview, see Barnett, Ceci [2002]). Acquired knowledge or skills may be narrowly specialized, like using such formalized procedure as applying the Rule of Three to solve proportions, or very broad, such as finding solution principles, heuristics, or deducing.

Thus, while modeling is built around the school mathematical language as an object, research on transfer employs an incomparably wider range of the types of content to be applied.

2.2. Application context

Not only does the transfer conception present myriad variants of the content of transfer, but it also explores various contexts, i. e. where an acquired skill is transferred from and to. The training and transfer contexts may differ along multiple dimensions (knowledge domain, physical and social context, temporal context (the elapsed time between training and testing phases), functional context (the function for which the skill is positioned), and modality (the final sensory dimension of transfer context)). The number of dimensions differing for the training and transfer contexts as well as the degree of similarity between the contexts determine the distance between them [Barnett, Ceci, 2002]. Near transfers have been found to be successful much more often than far transfers. In other words, transferring an acquired skill immediately to a structurally and formally similar problem in the same context will be much easier than transferring the same skill to a problem presented over time and contexts. The difficulty of far transfer is explained by its mechanism, i. e. the need to draw an analogy between the structures of two problems.

2.3. Process and mechanism

The mechanism of transfer is most often described as drawing an analogy and comparing the new case to the training one [Gentner, 1983; Gentner, Loewenstein, Thompson, 1999; Gick, Holyoak, 1980; Reed, 2012]. Transfer is normally regarded as a one-time event, but some researchers divide it into three processes: (i) remembering a prior analogous situation in long-term memory (retrieval); (ii) aligning the representations of two cases (mapping); and (iii) judging the adequacy of solution found for the new problem (evaluation) [Gentner, Smith, 2012]. The success of transfer depends mostly on the first two stages, where difficulty is determined by differences in formulation and contexts, and on how solvers encode problems.

When we encounter a new problem, we need to access a potential analogy, i. e. to retrieve a potentially known analogous problem from memory. Surface-level similarity plays the decisive role here. If there is no such similarity, it will be hard to access a previous problem even if it is stored in the long-term memory. This phenomenon is sometimes referred to as *inert knowledge* [Gentner et al., 2009], i. e. potential useful knowledge that is unavailable at the right time. “Inertness” of knowledge exists because people use context-specific ways of encoding their experience [Gentner, 1983], so the required knowledge is only activated through similar surface characteristics of problems: context details or items involved.

At the second stage of transfer, relations between elements in the training and transfer problems are compared [Christie, Gentner, 2010; Reed, 2012]. Based on the analogy with the training problem, a solver comes to an inference about solving the new case. To illustrate this, let us take the two classic problems in the research on transfer [Gentner, 1983]. The first (training) problem sets a military goal: a general wishes to capture a fortress and the only way to do it is with a full-scale direct attack, but it is impossible for a large force to pass to the fortress. The solution is to send the soldiers via different roads and converge them all simultaneously on the fortress. The new (transfer) problem describes a medical issue: a type of ray could be used to kill a cancerous tumor; however, in the dosages needed it would also kill the surrounding tissue. The approximate similarity patterns will be built as follows: soldiers → beams; fortress → tumor; narrow access routes → tissue destruction; military power → radiation dosage [Gentner, Smith, 2012]. This comparison produces an inference: multiple low-intensity rays should be simultaneously directed toward the tumor from different directions.

At this stage, the training problem solution method should be encoded at the abstract level as a principle of converging forces coming from different sources. Thus, a structural analogy with the transfer problem can be drawn. However, if the training problem is encoded at the level of surface features (tumor, X-rays), the analogy with the correct solution method will be unavailable when solving the transfer problem.

We can see that transfer success factors are obviously in conflict here: whereas the first stage requires surface-level similarity to retrieve a similar problem from memory, the second stage implies element-level similarity to find structural matches. Given that surface similarity often does not entail a structural one and may be misleading, the conflict becomes twice as significant, especially in the learning context, which we will dwell upon below.

At the third stage of transfer, the solver evaluates the goal relevance of the produced solution, the analogy and its inferences [Gentner, Smith, 2012]. The role of mental representation is minimal here, giving way to metacognitive skills, such as control.

2.4. Conducive learning

The learning approaches proposed by the modeling conception correlate poorly with the postulated mechanism of applying acquired knowledge in new situations. Contrastingly, researchers of transfer seek to connect logically the learning methods with the mechanism of transfer.

2.4.1. Understanding the abstract structure and the role of concrete examples in learning

As ample research proves, understanding the deep structure of training material is absolutely vital to enable correctness of using acquired knowledge in new contexts. It means that problems should be encoded at the abstract level, i. e. surface features of specific contexts should not be included as key elements in a mental representation. With abstract-level encoding, acquired knowledge can more easily be transferred to the most diverse concrete situations of new problems than with learning from a few concrete examples and associating solutions with specific surface details of a context.

At the same time, specific surface-level similarities allow for memorizing a similar problem solved in the past as a prerequisite for spontaneous transfer. In order to detect a surface-level similarity, one should be familiar with features of specific types of training problems. Abundant context features facilitate access to training problems when dealing with new ones, which increases the chances for a successful transfer. In this theoretical perspective, learning should be based on material rich in contextual features and detailed examples that will facilitate retrieval. Besides, examples also probably facilitate understanding of the problem structure. So, should training material be abstract or associated with possible contexts of potential applications?

This dilemma inherent to transfer postulates is supported by ambivalent experiment results (for a more extensive discussion, see, e. g. Reeves, Weisberg [1993]). On the one hand, it was shown that solution methods are more easily transferred from abstractions to concrete examples than vice versa. Students who had learned arithmetic progressions (algebra, abstract level) were very likely to recognize that physics problems involving velocity and distance can be addressed using the same equations. In contrast, students who had learned the physics topic almost never exhibited any detectable transfer to the more abstract isomorphic algebra problems [Bassok, Holyoak, 1989]. Qualitative studies also prove that understanding the abstract structure of a problem plays an integral role in the mechanism of transfer [Robertson, 1990].

The so-called schema-based instructions also confirm the effectiveness of abstractions for transfer. Studies demonstrate that schematic representation, i. e. identifying the problem schema, ensures better understanding of the underlying structure of the problem [Logie, 1995; Poltrock, Agnoli, 1986] as well as effective problem-related communication [Abdullah, Halim, Zakaria, 2014]. However, schematic representations only proved to be useful for solving difficult problems, whereas in simple problems it only increased cognitive load without facilitating the process. Researchers explain this discrepancy in schema effectiveness by saying that difficult problems require a bundle of operations, so schemas come in handy, reducing the short-term memory load [Beitzel, Staley, 2015; Zahner, Corter, 2010].

On the other hand, there are findings that confirm the need for "applied", context-rich problems and concrete examples for a good transfer. For instance, students who previously participated in the application exercise activities transferred statistical knowledge to real-life applications more successfully compared to students who did not do any application exercise [Daniel, Braasch, 2013].

More in-depth studies revealed that transfer performance is affected not by context-based training problems and examples as such but by their learning applications, their correlation with the abstraction under study and the degree of difference between them. Only instances compared to one another to derive a general schema for a class of instances facilitated forward transfer, compared to situations where instances were offered but no comparison was provided [Gick, Holyoak, 1980; Kurtz, Boukrina, Gentner, 2013]. It also turned out that a simple instruction to solving method is not enough, and neither are simple example-based illustrations. Only searching for similarities across instances or between an instance and the general schema increases the likelihood of forward transfer substantially [Gentner, Loewenstein, Thompson, 1999].

Examples that revealed links between the abstraction under study and required calculations facilitated application of the abstraction in new contexts, while examples and training problems designed to enhance calculation or procedural skills showed no positive effect on transfer [Catrambone, Merrill, 2003].

The extent to which contexts in training problems are different has contradictory effects on the learning process and forward transfer. Close similarity between contexts allows students to derive the problem structure but has no great impact on forward transfer. When contexts differ in a number of features, deriving the common structure consumes a lot of time and effort, but forward transfer is considerably enhanced [Didierjean, Nogry, 2004; Gick, McGarry, 1992].

On the whole, it appears that teaching abstractions and the abundance of instances in training material alone play no important role. Rather, what matters is student's activities aiming to derive the common principle from concrete context-abundant problems or to detect the worked abstraction in various detailed contexts.

The methods of deriving the problem structure, or the common principle, may be different. Apart from working with training examples, direct structure-deriving orientation and activities are also effective, like marking explicitly the subgoals of a complex math procedure [Atkinson, Catrambone, Merrill, 2003].

2.4.2. Learning attitudes

Many researchers agree that transfer performance depends heavily on the learning attitudes of teachers and students. This follows from the studies comparing the effects of constructivist and traditional classroom learning [Engle et al., 2012; Serafino, Cicchelli, 2003]; the studies where students are encouraged to identify conditions relevant for knowledge application on their own and to explain their ideas not only to their teacher or class but also to other people; the studies where potential extracurricular applications of what is learned are demonstrated and students have to deal with these "extended applications" on a permanent basis; the studies where students are allowed to correct their mistakes on their own, etc.

Another collection of studies have to do with developing an attitude to a specific type of cognitive work, i. e. establishing the habit of analyzing the structure of a problem before trying to solve it. One of the few studies in this line shows that students who previously solved a problem requiring analysis of interrelations are likely to interpret new problems from the same perspective [Bliznashki, Kokinov, 2010]. Ann L. Braun and Mary Jo Kane succeeded in developing the skill of searching for structural analogies between examples as a habitual thinking pattern in preschool children [Brown, Kane, 1988].

Transfer is also enhanced when students change their orientation from *performance* goals to *mastery* goals. *Performance* means succeeding in training tasks and demonstrating one's skills as compared to the rest of the class, while *mastery* implies achieving personal learning and self-development goals associated with long-term success. The change from *performance* to *mastery* goals in the motivational profile of students has a positive effect on various aspects of the quality of learning, including transfer. In a recent study on learning of negotiation strategies, two teams were given different instructions: for one, immediate achievement and error minimization were emphasized, while the other was instructed to master the material [Bereby-Meyer, Moran, Unger-Aviram, 2004]. No differences were found in near transfer. However, in problems with modified scenarios and new condition features (far transfer), the team primed with mastery goals performed better than the team primed with performance goals. Similar results were obtained in a study where students were given different goals: immediate goal achievement vs. free exploration of a problem in the absence of a specific goal [Vollmeyer, Burns, Holyoak, 1996].

2.4.3. Metacognitive skills

The skill of analyzing a problem to derive its underlying structure, which is so useful for transfer, is very close to a set of skills that are considered indispensable for solving any types of problems: critical thinking, self-reflection, control, planning, and introspection—usually referred to as metacognitive skills. There is every reason to expect that purposeful development of these skills will contribute to a successful transfer of acquired knowledge to new situations. Surprisingly, very few studies address metacognitive skills as a predictor of effective transfer. Yet, there is empirical evidence that the development of metacognitive skills actually enhances transfer, for instance, by encouraging reciprocal learning that promotes introspection and self-monitoring [Bransford, Schwartz, 1999].

The role of metacognitive skills is also confirmed by research on the effects of the learning programs involving self-explanation, i. e. explaining to oneself specific steps in the solution, discussing with oneself the goals, the results and the relations between consecutive actions. This research revealed that students with well-developed self-explanation skills elaborate a strategy to solve a problem instead of chaotically trying different ways to find a solution and that transfer skills are enhanced by using strategies. Michelene T. H. Chi and her colleagues [Chi et al., 1989] found out that students who used self-explanation in learning performed much better in transfer-related problems. Self-explanation represents a two-fold mechanism, integrating new information with relevant inferences and helping students detect and repair any inconsistencies between the constructed mental model and the proposed problem situation [Chi, 2000]. (For similar studies providing a more in-depth analysis of the effects of different self-explanation techniques under different problem conditions, see Renkl et al. [1998]; Atkinson, Catrambone, Merrill [2003].)

Table 1. **The structure of knowledge application in modeling and transfer**

Object (what is applied)	Modeling	Transfer
	Mathematical language	Varies greatly
Context (where it is applied)	In theory: real-life situations. In practice: mathematical word problems	Varies greatly
Mechanism/Process (how it is applied)	The procedure: 1) structuring (constructing a situation model); 2) mathematizing (encoding the situation model in the mathematical language and transforming it into a mathematical model); 3) model manipulations, calculations; 4) interpreting (judging) the result	The procedure: 1) retrieving a similar problem from long-term memory; 2) drawing a structural analogy, comparing, aligning, drawing inferences; 3) judging the constructed analogy
Conducive learning	Learning conditions favoring effective modeling are only described in general terms because modeling as such is considered part of a broader culture of "conscious" teaching of mathematics. Such conditions, in particular, include: understanding the importance of modeling by both teachers and students; creating a "modeling discourse"; teaching the modeling strategy directly to students; allowing students to establish correlations among different mathematical topics as well as between mathematics and real life on their own; metacognitive skills; a constructivist framework in learning	• comparing concrete examples so as to derive the underlying structure shared by problems; • developing transfer orientation; • developing problem analysis focused thinking; • metacognitive skills of analysis, reasoning, self-assessment, self-explanation, etc.

3. Conception convergence: the key cognitive steps toward knowledge application

To summarize what we have said about the nature of transfer and modeling and about the methods of providing conducive learning environments, we reduced the above discussion to a table (Table 1). The first fundamental difference between modeling and transfer is the relative narrowness of the former and wideness of the latter. The modeling construct has to do with mathematical language application as a school subject and its only educational outcome. The scope of application is said to embrace all sorts of real-life situations, but in fact, modeling skills are learned from mathematical word problems representative of real-life situations. Transfer is described as the result of any type of learning, and the scope of application is also unlimited in theory. From this perspective, the transfer conception looks more promising for achieving the learning goals than the modeling conception. However, there have been few studies on the process of transfer, so additional effort is required to develop the relevant learning technology.

There is a certain similarity between the two conceptions in terms of describing the mechanism of knowledge application. Both modeling and transfer imply constructing a situation model for the new problem. In both conceptions, the situation model serves to structure a specific situation in more general, abstract terms. In fact, it represents a common description of all isomorphic problems, as it includes no surface features specific to each individual problem. Both conceptions approach this step as crucial for providing successful knowledge application.

However, the rest of the process differs: in transfer, the solver compares situation models of the training and transfer problems and decides on the possibility of solving the latter one; in modeling, themodeled relationsbecome even more abstracted and the general terms describing the situation model are re-encoded into even more abstract mathematical symbols.

Mathematical models are only used in the modeling conception, while there is no such element in transfer. A mathematical model allows for establishing precisely the quantitative relations in the situation model. In addition, a constructed mathematical model makes it possible to express one problem structure element through another, to assess how changes in one value affect the dynamics of another, and to perform other mathematical operations. Not only mathematical model manipulations allow for solving a new problem, but they are also used to make predictions and find boundary conditions for all isomorphic problems.

Both modeling and transfer place paramount importance on the possibility of making representations, different in the degree of abstractness but still synonymic. "Synonymic" means that all the key elements of the source concrete problem can always be found in the statement, whatever the level of generalization. In modeling, we have a problem (a real-life situation) and several steps of translating it into an ultimate abstraction to construct a mathematical model and back to interpret the mathematical solution into the real-life context. In transfer, there are two conventional levels of statement, concrete and generalized, and one transformation, i. e. construction of a situation model. At the same time, each transformation in any of the conceptions suggests that structural consistence between the levels of statement should be maintained (provided).

Differentiating between the levels of abstractness/generalization is purely conventional, just as the levels themselves are. It only serves to show that a situation model is described in more abstract terms than a concrete situation, and a mathematical model is more abstract than a situation model. In fact, we would prefer addressing them as an uninterrupted transition in the *concrete—abstract* continuum than as discrete levels.

Our analysis shows that constructing a structural statement of a problem (situation model) is a crucial knowledge application step in both modeling and transfer. Another indispensable procedure in both conceptions is structural comparison, i. e. establishing consistencies between situation statements of different degrees of generalization. Based on this idea of prerequisites for adequate knowledge application, we can reevaluate the conducive learning practices proposed by each of the conceptions. First, everything associated with developing the right motivation and attitude may be interpreted as nonspecific assistance to activate cognitive activities as such, structuring being an isolated case of these activities. Second, the greatest importance among metacognitive skills belongs to analyzing, comparing and generalizing, which form the basis for structuring and structure comparing. Third, the key role is played by direct instructions to compare superficial differences though isomorphic problems and then abstract the structure they share.

Structuring and comparing as the fundamental prerequisites for transfer may be used as training problem assessment criteria. If students engage in comparing processes or situations represented at different levels of generalization or in different symbolic systems (text—diagram—function), or if they transform one representation into another maintaining the structural consistency, such work is expected to promote the formation of highly transferable knowledge.

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