Analysis of Real-World Math Problems: Theoretical Model and Classroom Applications

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Abstract. The Russian education standards stress the importance of real-life applications of mathematics. However, the educational outcome standards do not provide a clear idea of how a math teacher should organize their syllabus to develop relevant skills in students. As long as there is no universal definition of a real-world math problem, it is rather difficult to qualify the problems that teachers use in the classroom. We analyzed algebra problems that teachers give to secondary school students. Using three parameters, 83 word problems were coded: situational relevance, mathematical modeling, and non-triviality. We carried out a cluster analysis to identify typical categories of mathematical problems. As a result, we determined three types of problems differing in the abovementioned characteristics. Only one cluster appeared to feature all three characteristics typical of real-world problems. Therefore, a portion of the tasks that teachers give students as real-world fail to qualify as such according to the proposed theoretical model.

Keywords: secondary school, algebra, real-world problems, everyday context, math word problems, transfer of learning, mathematical modeling.

Education standards in many countries emphasize the vital importance of bridging school mathematics with real-life situations. A Realistic Mathematic Education approach was developed in the Netherlands in the mid-1970s. Later on, many countries picked up the idea and continued research in this area [Treffers, 1993]. The Mathematics in Context curriculum was launched in the US and the UK in 1996 [National Council of Teachers of Mathematics, 2006; Dickinson et al., 2011]. In 1997, Norway also introduced new education standards which regarded teaching "everyday" mathematics just as important as teaching arithmetic, algebra and geometry [Royal Ministry of Education, Research and Church Affairs, 1999].

National education standards in Russia were amended in 2010, and they also placed a focus on developing knowledge application skills, in line with the global trend. Thus, the 2010 Federal State Gen-

eral Education Standard imposed the following requirements for the school mathematics curriculum: "By learning the subject area of Mathematics and Informatics, students should realize the importance of mathematics and informatics in everyday life". As for mathematics education outcomes, they should demonstrate "the ability to model real-life situations in algebraic language, to analyze the constructed models using algebraic tools, to interpret the results obtained" and "the ability to apply worked concepts, results and methods to solving real-world math problems and problems in allied disciplines".

While education standards determine requirements for education outcomes, The Fundamental Nucleus of General Education Curriculum Content (2011) should be applied to the development of curricula, syllabi, teaching and learning aids. This document clarifies the conception of the new standards, defining the goal of learning mathematics as follows: "Mathematics helps solve real-world problems: family budget optimization, time management, critical evaluation of statistical, economic and logical information, correct assessment of partnership/offer profitability, and simple engineering and technical calculations required in real life".

Therefore, the two key national documents regulating the content of education in Russia underline the importance of applied methods of teaching mathematics in school. However, the abovementioned requirements for mathematics education outcomes do not provide a clear idea of how a math teacher should organize his or her course to develop applied math skills in students.

The new standard also changed the assessment and testing materials. Today, the math exam for Grade 9 students consists of three modules: Algebra, Geometry and Real Mathematics. The Specification of Assessment and Testing Tools for the 2015 Basic State Examination in Mathematics defines Real Mathematics problems as "problems that are worded to contain a real-life context familiar to students or close to their life experience". Explaining the goal of such problems, the authors use phrases like "real numeric data" and "real relations between values". However, the Specification does not stipulate any criteria for qualifying a problem as having a real-life context or being associated with personal experience, and neither does it provide insight into the concept of "real data". That is why the specificity of Real Mathematics problems still remains unclear despite the available definition.

Similar changes occurred to the math exam for Grade 11 students. Now, the Specification of Assessment and Testing Tools for the 2015 Unified State Exam in Mathematics prioritizes the goal of assessing the ability to "apply acquired knowledge and skills in realistic contexts and everyday life". Consequently, the Unified State Exam (USE) now includes problems aimed at "testing the basic expertise and skills of applying mathematical knowledge in real-life situations". However, this definition does not establish any criteria for identifying problem wording as a "real-life situation" either.

Solving problems of this type successfully contributes quite a lot to the final score in both tests. Thus, the Real Mathematics module in the BSE consists of seven problems out of a total of 26. At least two of these seven problems should be solved to get the minimum passing score. The USE offers four (out of 20) problems to test the ability to apply acquired knowledge, their contribution to the initial test score being 20%.

Teacher attitude to transformations is what largely determines the success of any education reform [Thompson, 1992; Barlow, Reddish, 2006; Handal, Herrington, 2003; Hanley, Darby, 2006]. Misinterpretation of the main ideas may become the biggest setback for reform implementation [Ross, McDougall, Hogaboam-Gray, 2002]. Discrepancies in understanding the concept of real-world problems by the school community can be a hindrance for introducing the new education standards in Russia. In the absence of a universal definition, teachers are guided by their own criteria when selecting real-world problems. For instance, a survey of 62 secondary school math teachers showed that they prioritized topical relevance over authenticity when choosing real-world problems for a lesson [Gainsburg, 2008].

This paper seeks to analyze the teacher's perception of the problems that allow assessing real-life math skills. Our research is aimed at answering the following questions:

- What are the qualification criteria for real-world math problems?
- What real-world problems are used by math teachers in the classroom?

1. The notion of a real-world math problem and its key characteristics

Real-world math problems are used in school lessons and final tests throughout the world. Yet, there is still no universal definition for these types of problems. Depending on the conception, researchers may call them real-world or realistic problems [Cooper, Harries, 2005; Gainsburg, 2008; Pais, 2013], modeling tasks [Blum, Borromeo Ferri, 2009; Frejd, 2012], contextualized tasks [Carvalho, Solomon, 2012; Palm, 2006], everyday problems [OECD, 2013], applied tasks [Palm, 2006], etc. In Russia they are traditionally called practice-oriented tasks, in particular when referred to in the specifications of assessment and testing tools. In this study, we use the term *real-world problems* [Fridman, 1977].

Definition and theoretical model determine the goal and the structure of real-world problems, so they are crucial for problem construction. Besides, only a theoretical model allows the authors to verify the conformance of problem realization to the project, i. e. the construct validity of a real-world problem. A theoretical model should include a set of universal criteria to compare and evaluate real-world problems in various application contexts. This overview aims to identify the set of characteristics common for real-world problems.

There are multiple definitions of real-world problems, which means that many different characteristics may be used in theoretical models of such problems. Using everyday language is the first criterion considered by all researchers: problems should describe situations with the help of words, symbols and events that people come across every day. However, everyday problems can be set in myriads of contexts, so it is important to identify clearly the characteristics of everyday situations to qualify problems based on those situations as real-world.

The Programme for International Student Assessment (PISA) sets real-world problems in three types of real-life contexts [Watanabe, Ischinger, 2009]. Problems with personal contexts have a direct reference to students' day-to-day activities, like buying a commuter rail ticket, going shopping or reading a package insert. Problems are also constructed in educational and occupational contexts. These situations are not restricted to everyday activities, and the content of relevant problems may be related to other school subjects, such as biology, chemistry, geography, etc. Finally, a real-world problem may require working with public information from newspapers, magazines, TV shows and the Internet.

Regardless of the type of context, it is vital to consider the degree of realism of the situation: it can be cases from personal or social life with real names and events, or it can be problems with fictitious contexts that have nothing to do with real life. For instance, the statistics course uses a number of problems asking students to find the probability of picking a black or white ball from a bag. Naturally, a situation like this is irrelevant to a student's life [Palm, 2006]. A special place belongs to the so-called cleaned contexts, which do not contain details or circumstances that are not required to solve the problem [Debba, 2011; Du Feu, 2001].

Real-world math problems are designed to make students apply conceptions and procedures that they have learnt from the school course. A problem situation dressed up in everyday language is supposed to be translated into mathematical language. Modeling, or identifying the relations between objects in a problem, is an important stage in solving any word problem, including real-world problems [Talyzina, 1988; Fridman, 1977, Blum, Niss, 1991]. The answer obtained has to be assessed with regard to the original context, i. e. interpreted back to that context. Therefore, the use of everyday language as the first characteristic of real-world problems implies two unavoidable processes: mathematical modeling and interpretation.

The second criterion of a real-world problem is situational relevance of context: objects and relations in the context should be directly relevant to the solving strategy and to the answer obtained. Another taxonomy of real-world problems was developed for PISA, based on the degree to which the context should be used to solve a problem [Watanabe, Ischinger, 2009].

1. Zero-order context: the problem is constructed using everyday language, but the context does not have to be used to find the solution.
2. First-order context: the context is relevant and needed for solving the problem and judging the answer. (Unlike in second-order contexts, the relations between objects are pre-modeled here, e. g. presented in a graph or formula.)
3. Second-order context: the context is also needed for solving the problem and judging the answer, but the student also has to construct a mathematical model of the situation in this context.

Thus, we can see that a problem designed to test the ability to use one's knowledge in real life should be (i) formulated using everyday language and (ii) situationally relevant.

Other researchers also point to the parameters describing the solving environment [Palm, 2006]. Specific conditions of the solving environment may contribute or, vice versa, detract from the solving process. Real-world problems should be constructed with due regard to the following: (i) availability of additional tools; (ii) availability of solving instructions; (iii) possibility to get advice or discuss an issue; (iv) time limit / no time limit.

Yulia Tyumeneva [2014] explored the key characteristics of real-life situations that should be preserved in real-world problems. In addition to everyday language and situational relevance, she also offered non-triviality and relative structural rigidness as real-world problem criteria. Non-triviality is understood as the unorthodoxy of a problem, i. e. the absence of any reference to the algorithm sought for [Jonassen, 1997].

Besides, as an analogy with real life, a problem cannot have a rigid structure or only one right solution a [Ibid.]. However, the school has to put limitations on the answer judgment criteria in order to provide reliability of judgment. That is why relative structural rigidness is regarded as a possibility of using more than one solving strategy.

Thus, an analysis of the existing conceptions allows us to identify the following parameters of real-world problems: use of everyday language (requiring modeling and interpretation), situational relevance, solving environment, non-triviality, and relative structural rigidness. These can be included in the universal theoretical model and serve as the basis for the construction of real-world problems.

Certain constraints are placed on the format and content of any task by the context of application. For example, mass testing does not provide a possibility of discussing the solving process or consulting an expert. In this case, it is very hard to ensure complete congruence of a real-world problem with reality in terms of the solving environment. Approaching this challenge at a broader level, we will also face the conflict between construct validity and test reliability. The maximum possible congruence of a task with real life would require

a complex and sophisticated assessment procedure and thus would work against any possible standardization and test reliability in general [Wiggins, 1993].

In our study, we analyze problems that math teachers use in the classroom, not those given in tests. The classroom environment in a regular school also places constraints on applying all of the real-world problem parameters. For instance, only direct observations show whether students are encouraged to interpret the results obtained. In fact, participant observations of teacher and student behavior are the only way to find out whether teachers require the translation of results back to everyday language and motivate students to use diverse and unorthodox algorithms.

The situation in which real-world problems are solved is often impossible to evaluate. Was there any time limit? Was the student allowed to consult an expert or their classmates? Was the student allowed to use additional tools? Specific constraints on congruence of real-world problems with day-to-day contexts are also imposed by the solving environment. Hence, the theoretical model of real-world problems that we have elaborated needs to be revised each time to meet the conditions and application goals at any given moment.

2. Research method

In the course of our research, we analyzed the problems that algebra teachers gave their Grade 8 and 9 students. The teachers were asked to do a demo lesson on a curriculum-relevant topic. They were instructed that "the lesson should be primarily designed to make the students understand the relations between what they learn in the classroom and possible applications in real life".

The study involved 18 math teachers in municipal general education institutions in eight regions of Russia. The students who participated in the demo lessons were learning under the basic program.

We sampled 83 word problems out of the tasks used by the teachers in the demo lessons. The teachers qualified those word problems as real-world and used them in the classroom to illustrate how mathematics could be applied to everyday life. Analysis of the problems did not consider the data obtained from observations of teacher and student behavior, which placed certain constraints on the research results.

We analyzed the selected word problems in two steps. First, we defined the parameters of the real-world problems with regard to the sample constraints and coded all the problems using those parameters. Second, we used a cluster analysis to analyze the resulting data matrix. We chose this statistical analysis method because it allows observations to be merged into homogeneous clusters based on the preset characteristics. In other words, a cluster analysis makes it possible to find out which types of real-world problems math teachers use in the classroom.

3. Problem coding   Given the goals and application contexts of the sampled real-world problems, our analysis was based on three theoretical model parameters: use of everyday language (only at the level of mathematizing), situational relevance and non-triviality.

1. Situational relevance. This parameter shows the relevance of the problem context to the student's everyday life. The context can make a reference to day-to-day activities, education or human-society interactions. The problematic nature of a question suggests that the solution can serve as the basis for specific actions in the given context.
Let us analyze the following problem as an example.

How many ways can Boldino Farm Firm sow rye, wheat, barley and corn on four ploughed fields during spring sowing?

The farm firm's activities make the context of this real-world problem. However, the question posed is not problematic as the decision to sow specific grass on a ploughed field has nothing to do with the variety of crop combinations.
The context of the following problem is situationally relevant because the solution can serve as the basis for specific actions.

A patient takes 5 drops of medicine on the first day and increases the dosage by 5 drops daily. As she reaches the dosage of 40 drops/day, she keeps to it for 3 days and then starts decreasing the dosage by 5 drops daily down to 5 drops on the last day. How many bottles of medicine should the patient buy if each bottle contains 20 ml of medicine, which is 200 drops?

2. Mathematical modeling. This parameter implies the need to translate the problem conditions from everyday language into mathematical language, e. g. to express the relationship between objects as an equation. The following problem is solved by presenting the numbers in the text as terms of a sequence and applying a formula to calculate the sum of the arithmetic progression.

A free-falling body travels about 5m in the first second and 10m more in each consecutive second. Find the depth of the mine if the body reaches the bottom in 5 seconds after it starts falling.

In some cases, problems formulated in everyday language do not require application of mathematical conceptions or procedures, e. g. when the student is supposed to work with graphs, charts or figures. Although such problems do ask the student to find a specific regularity, no special mathematical knowledge is needed to do this, so translation into mathematical language is not required.

Figure 1. **Heating curve for water**
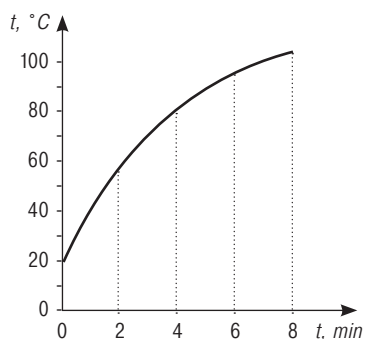
The graph in the figure shows the heating curve for water based on data obtained by a student. Answer the following questions: What was the temperature of the water when the countdown began? By how many degrees did the temperature of the water change in the first 4 minutes? By how many degrees did the temperature of the water increase in the last 2 minutes of observation? (Fig. 1)

3. Non-triviality. We understand non-triviality as the absence of reference to routine types of problems, i. e. the absence of any clichéd wording that gives a hint of the specific solving algorithm. The following problem contains routine language that sends the student to the required strategy.

A tiler lays 3 tiles in the first row, 5 in the second row, and so on, 2 tiles more with each row. How many tiles will he need to lay the seventh row?

This problem suggests applying the arithmetic progression formula and can be found in school textbooks quite regularly. Whereas, in the problem on buying the right number of bottles of medicine described earlier, the formulation of problem conditions and question is not connected with any specific topic in the curriculum.

Each parameter of our theoretical model is a dichotomous variable, coded as 1 if present and 0 if absent. All of the selected problems were coded by the same person based on the three parameters mentioned above. To test the reliability of the developed theoretical model, three independent researchers were offered to code 25% of randomly sampled problems. The experts made coordinated decisions in 90% of the cases, which is an acceptable level of the coding system reliability.

| 4. Clusters of real-world problems | Out of the total sample, 63% of the problems were coded as requiring mathematical modeling. Situational relevance turned out to be typical of only 26%. Only 12% of the problems had non-routine wording. The specific nature of our research did not allow any assessment of the content of real-world problems. |

We applied hierarchical clustering to identify the types of real-world problems. This choice of method was made because typing is a search-driven objective and the number of clusters is not predetermined. In hierarchical clustering, each observation is treated as a separate cluster and then merged with the closest similar observation into another cluster. Clusters merge cases that are the closest to one another based on the specified characteristics. The clusters themselves represent groups of observations that differ from one another as much as possible based on the same characteristics. This type of analysis identifies the existing categories of problems, but not the relations between observations, variables or clusters.

The cluster analysis was performed using the SPSS20.0 statistics package. Assuming that clusters can be a different size, we chose the within-groups linkage method for clustering. As the clustered variables were dichotomous, we measured the distance between observations using the Sørensen-Dice coefficient.

As a result of hierarchical clustering, we classified observations into three groups. The identified clusters differ from one another in terms of the specified real-world problem parameters that served the basis for clusterizing. Hence, this classification describes the types of presented problems in the best possible way.

Table 1 shows the number of objects in each cluster and their percentage share in the overall number of observations. The resulting clusters differ in the number of observations in them, and this decision seems adequate as the sample is not representative of the different types of problems. We did not control the sources that teachers could use to select problems for the lessons. It might be that some types of problems were easier to find than others. Therefore, the equal size requirement appears to be unrealistic.

As we can see in Figure 2, the problems grouped in each cluster are characterized by different sets of parameters.

Cluster A groups real-world math problems, each of which is non-trivial and situationally relevant. However, no mathematical modeling is required to solve these problems. They can be formulated using everyday language, and the question may be relevant for the given context, but solving the task does not imply the application of mathematical knowledge.

Cluster A mostly contains problems on working with graphs and charts, it also includes the water heating problem (Fig. 1). The problem is worded in the context of an experiment, which is familiar for the learning process. However, solving the problem implies reading a pre-constructed graph, so mathematical modeling is not required.

Table 1. **Distribution of problems across clusters**

	Observations in a cluster	Share in the total sample (%)
Cluster A	7	8
Cluster B	52	63
Cluster C	24	29
Total problems	83	100

Figure 2. **Percentage distribution of real-world problems across clusters**

Cluster A

%
100
0
100

Cluster B

2
100
0

Cluster C

29
100
100

- Non-triviality
- Mathematical modeling
- Situational relevance

Most problems in *Cluster B* have only one parameter: they require mathematical modeling. Only 2% of the problems are also qualified as non-trivial, but this insignificant proportion can be reasonably neglected. The problems in this cluster are worded in everyday language and require translation into mathematical language. Meanwhile, they are not situationally relevant, and they are also formulated using routine words and phrases pointing to the specific solving algorithm. Let us give an example.

> Two cyclists started a 60km ride at the same time. One cycled 10km/h faster than the other and finished the ride 3 hours earlier. What was the speed of the cyclist who came second?

The context of this problem is trivial and also familiar to students. However, the question it sets is not problematic and the solution cannot be considered important for the student. Moreover, the very description is routine: it can be easily found in math textbooks and requires a specific, well-known solving method.

Cluster C is represented by the problems which are all situationally relevant and suggest mathematical modeling. However, only 29% of the problems in this cluster are formulated without using phras-

es referring the student to a specific solving paradigm. The following problem is typical of this cluster, and it has all three of the specified parameters.

> A park has a railway loop line and a bike path, the movement along which is performed according to the equation $y=0.16x^2-32x+1,300$. Locate traffic lights to ensure the safety of cyclists.

This problem is solved with the help of graphs. The situation it describes is rather common in park planning, thus it is situationally relevant.

5. The teacher's choice of real-world problems

Math teachers now have to pay more attention to real-world problems due to the introduction of new national education standards and the associated changes in the content of final tests. However, they find it difficult to select problems for the classroom in the absence of clear qualification criteria. This study was performed to summarize teachers' ideas of real-world math problems and to determine the most widespread types of such problems.

As a result, we identified three categories of real-world problems which differ greatly from one another within the theoretical model that we developed. Thus, problems in one cluster have only one real-world problem parameter, namely they require the translation of problem conditions from everyday into mathematical language. Meanwhile, this parameter is not typical of any problem in another cluster. There is only one cluster where word problems possess all the three qualification characteristics. It means that some of the tasks that teachers use as real-world problems fail to qualify as such.

Problems of all the types identified can be used in math lessons, but teachers should not forget about the main goal of real-world problems, which is to test the student's ability to apply mathematical knowledge to real-life contexts. That is why it is extremely important for a real-world problem to be as congruent as possible with a real-world situation, i. e. to have the key characteristics inherent to this type of problem.

A math teacher decides which problems should be solved in the classroom, which of them are suitable for covering a specific topic, and which develop required competencies and skills. The characteristics of real-world problems are refracted through a prism of the teacher's attitudes. As we showed in our study, teachers may sometimes prioritize mathematical modeling or non-triviality as the key qualification criterion in different situations. The results that we obtained are consistent with the findings of another research, which revealed that teachers underestimate the value of real-world context authenticity [Gainsburg 2008].

The absence of clear qualification criteria of real-world problems also results in the lack of sources where teachers could find such tasks; no guidelines or manuals with real-world problems are provided for math teachers. In addition, teachers have to adjust real-world problems to specific learning situations. For instance, some mathematical conceptions are difficult to explain using real-world problems, but abstract material does not allow teachers to demonstrate real-world applications of mathematical knowledge. The teacher may simplify real-world problems by reducing the "contextual noise" and emasculate their everyday-language content to the benefit of standardization. On the one hand, teachers are faced with the necessity to develop problems on their own; on the other hand, independent problem construction may compromise the validity and reliability of real-world problems [Wiggins, 1993].

The sampling method that we used places some limitations on the research. We only analyzed the texts of real-world problems, without considering the results of classroom observations. Having not analyzed how teachers work with such problems, we cannot say whether they asked the students to translate the results back to everyday language or whether they allowed the students to choose a solving strategy, a parameter that we call flexibility. Therefore, the parameters of interpretation and flexibility were dropped from the theoretical model.

The sampled problems were used by math teachers during demo lessons. It is not improbable that they work with other types of tasks in their everyday practice. Besides, we cannot judge on the representativeness of these types of real-world problems all over the curriculum for Grade 8 and 9 students.

In summary, specific aspects of research design leave us with no data to make some important conclusions on math teachers' practice of using real-world problems in the classroom. Observing how teachers and students deal with these problems, as well as possible standardization of this process, are promising paths of research in this field.

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