

Comparative Study of Secondary School Mathematics Teachers' Beliefs and Practices in Russia, Estonia and Latvia

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Abstract. Teachers' beliefs should be changed in order to introduce modern teaching methods in education. The notion of "belief" combines the ideas, attitudes, and personal philosophies teach-

ers apply in their work. We differentiate between traditional beliefs about teaching as a direct transfer of knowledge and constructivist beliefs assuming that students construct their knowledge themselves through specifically organized activities. We have analyzed the key teacher belief research projects: the OECD's Teaching and Learning International Survey (TALIS), the cross-cultural Teacher Education and Development Study in Mathematics (TEDS-M), and the Nordic-Baltic Comparative Research in Mathematics Education (NoRBA) that we borrowed a questionnaire from. Our survey involved teachers of mathematics in three countries: 390 teachers in Latvia (of which 95 with Russian as their native tongue), 332 teachers in Estonia (of which 92 with Russian as their native tongue), and 1,096 teachers in the Russian Federation. We have found that differences between teachers in different countries were statistically important in all the variables used in the study, regardless of whether Estonian and Latvian teachers were Russian-speaking or not. All teachers implemented their beliefs in their everyday classroom practices. 36% of teachers in Russia had a high level of constructivism (as compared to 26% in Latvia and 18% in Estonia). Proportion of teachers with low levels of traditionalism in Latvia and Estonia (appr. 25% in both) was higher than the same proportion among Russian teachers (17.5%). We have come to a conclusion that different approaches to edu-

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cation system reforms in Russia and in the Baltic states have resulted in a significant diversity of beliefs among teachers of mathematics. Thus, proportion of teachers with low levels of traditionalist beliefs has grown in the Baltic countries more than in Russia, which explains to

some extent higher PISA points of Estonian and Latvian students.

Keywords: school education, mathematics teachers, beliefs, traditionalism, constructivism, PISA, Nordic-Baltic Comparative Research in Mathematics Education.

International comparative studies show that quality of school education differs from country to country. Clearly, the great variance of test results is caused by a number of reasons, but quality of teaching has an immediate and powerful influence on students' academic achievement. Of course, educational systems in countries with consistently high ratings in international studies are different in the structure and content of their education, but all of them are focused on enhancing the quality of teachers' work.

In modern school, teachers of mathematics play a very important role: mathematics is an essential tool of learning, and mathematical knowledge forms the basis of the whole economic theory.

Early works devoted to mathematics education were mostly centered around mistakes and difficulties encountered by students. Later, the focus of research gradually shifted towards investigating mathematics education as a system of three elements: students, curriculum, and teachers. Now, in particular, fields of research include correlation studies about teachers' goals and other formal characteristics, like teachers' experience or years of teaching, and students' achievements.

Investigations of professional math teacher qualities have been conducted in two main directions: 1) study of professional competencies (subject and pedagogical knowledge); 2) study of attitudes and beliefs. Particularly, the related works have discovered that beliefs of math teachers affect the process of teaching significantly [Thompson, 1992; Schoenfeld, 1998] and serve as a link between the cognitive (knowledge-related) and behavioral (teaching practices) components of teaching: "Belief is a bridge between knowledge and action" [Schmidt et al., 2007].

In this paper we compare beliefs of secondary school math teachers in three countries: Russia, Estonia and Latvia¹, with an emphasis on comparing beliefs of Russian-speaking teachers of all three countries.

¹ This paper relies on results of the project "An In-Depth Study of the Results of an Assessment of Education Standards in the Context of Economic Indicators in Education" carried out as part of the HSE Program of Fundamental Studies in 2013.



Secondary school provides mathematical knowledge required in everyday life and builds basic thinking skills that can be applied in a lot of professional fields. Secondary school allows students to master mathematics, relevant in their day-to-day life and develop a mathematical foundations that they can use in many other professional fields. It is the secondary school where the ability to construct and articulate mathematical proofs and arguments, to perform calculations and treat the data, to detect mathematical errors is developed.

That is why secondary school mathematics teachers were chosen for investigation in our study.

The choice of teachers from Russia, Estonia and Latvia for comparison has a number of reasons behind it. First, these countries still have much in common, as a lot of teachers earned their diplomas in the Soviet era and have been keeping to traditions. Second, some of the current trends in development of educational systems are common for the three countries: attractiveness of the teaching profession gets ever more undermined; low pay and extremely poor career opportunities make the best teachers of mathematics and students of teacher training colleges look for non-school jobs. Third, natural sciences and mathematics used to prevail in Soviet curricula, but later they ceded the spotlight to other fields of science. The teachers who didn't quit the profession have had to adapt to less motivated students, to the new learning programs, to the increasing cuts in hours of mathematics, etc.

Educational systems of all the three countries have undergone substantial changes since 1991. Latvia has introduced new standards of primary and secondary education which reinvented the philosophy of Latvian education, placing its main focus on providing students with knowledge and skills needed in everyday life [Sapkova, 2011]. Russia has also developed new national standards of education which prioritize distribution of activity-based (project, research) teaching practices to keep students interested in learning throughout the whole process and to develop leadership qualities, independent thinking, and ability to cooperate with others. Estonian researchers are worried about the syllabi being dominated by exercises and tasks designed to make students overlearn and drill specific patterns or algorithms. A learning approach like that certainly enhances performance but at the same time leaves children unsure of their chances of mastering mathematical knowledge [Lepik, 2005] .

Traditions of high-quality mathematics education are still persistent in Russia, which is demonstrated by high TIMSS scores of Russian students in natural sciences and mathematics (Russian 8th graders ranked sixth out of 43 countries in the 2011 study) and successful performance at international mathematical competitions for gifted and talented children. However, achievements of 15-year-old Russians in PISA are much less prominent: Russia placed 31st out of 65 countries, a 2012 study revealed a huge lag behind students from 30

countries out of 65. While being among the best in the TIMSS mathematical test, Russian school students lose to their peers from most other developed countries in PISA in terms of the key parameters of functional literacy, as their ability to apply knowledge and skills in practice is underdeveloped. Estonia demonstrates considerably higher PISA results in mathematics than Russia, and Latvia also out-scores Russia in this test. Our study is designed particularly to contribute to identifying possible reasons for such differences in performance of students from the three countries.

**1. Teachers’
Beliefs—What
Are They
About?**

The First European conference on Research in Mathematics Education held in Osnabrück in August 1998 proposed the following synonyms for the term “beliefs”: conceptions, attitudes, knowledge, practices, visualities, metaphors, views, perspectives, values, implicit theories, personal theories, personalized ideas, the Self, rules of thumb, frames, frameworks, outlook. From our point of view, “beliefs’ as a term is deeper and broader and also includes unconscious, implicit knowledge, views, attitudes, etc.

Noddings emphasized the role that research of teachers’ views and beliefs concerning mathematics and the way it’s taught played for understanding mathematical behavior of teachers and students [Noddings, 1990]. Beliefs research is especially important to change working practices when innovations affect accepted standards, programs or requirements to teaching techniques. This is how constructivism inspired a new branch of research: research of views and beliefs of mathematics teachers and students.

Rokeach defined beliefs as “any simple proposition, conscious or unconscious, inferred from what a person says or does, capable of being preceded by the phrase “I believe that”” [Rokeach, 1968. P. 2]. Works devoted to research of math’s teachers beliefs and students have been published since the mid-1980s [Thompson, 1984; 1992; Frank, 1988; Garofalo, 1989; Underhill, 1988]; however, no consensus has yet been reached regarding the scope of beliefs as a concept.

Pajares attempted to synthesize research in this field and to “clean up a messy construct” in 1992. He summarized the existing studies on teachers’ beliefs (not only in mathematics) and came to the conclusion that there are no specific beliefs; teachers’ beliefs are indissolubly interconnected. In their professional practices, teachers rely upon a whole system of views which, in turn, are based on deeply-rooted beliefs. That is why beliefs research requires that we don’t classify or differentiate between beliefs but try to extract the common teacher-specific understanding of mathematical education [Pajares, 1992] .

“Beliefs’ is a fundamental concept understood as a regulating system of the knowledge structure [Pehkonen, Törner, 1995. P. 1]. Beliefs occupy the transition zone between the cognitive and the affective,



bearing features of both. They represent a rather stable subjective (experience-based), implicit knowledge of an individual about mathematics and its teaching/learning. Pehkonen and Törner define conceptions as conscious beliefs, differentiating them from deep beliefs that are often unconscious. An individual's belief system is intertwined with their knowledge system so closely that the two systems are hard to analyze in isolation from each other [Pehkonen, Törner, 1995. P. 2].

Furinghetti and Pehkonen distinguish between objective and subjective knowledge and refer beliefs to subjective knowledge with affective components [Furinghetti, Pehkonen, 2002] .

In this paper, we are going to apply the term in a quite broad sense, understanding beliefs *as conceptions, views and personal ideology that teachers hold in their practice.*

Numerous studies have revealed that teachers' beliefs are formed by school practices, including both learning experience as a student and influence of colleagues and the environment [Pehkonen, 1994]. R. Philipp has noticed that beliefs are often inconsistent with the teaching practices teachers stick to, which he blames on the "context", i. e. various constraints, such as lack of time or possibilities, working conditions, specifics of some programs or requirements for teachers, student behavior, etc. [Philipp, 2007]

To introduce innovative teaching practices, teachers' beliefs should be reshaped both in teacher education universities and in advanced training classes.

Being rather stable psychological states, beliefs have been proven to gradually change under certain conditions [Törner, 2002. P. 117; Kaasila et al., 2006; Kislenco, Lepmann, 2011]. Superficial, freshly-formed beliefs are particularly easy to reshape [Pajares, 1992] .

American researcher A. Thompson developed a three-level framework of the development of teachers' conceptions of mathematics teaching² (cited by [Pehkonen, 1994. P. 194]) .

Level 0 of the framework is characterized by understanding the nature of mathematics as purely calculative. In fact, this is about machinelike teaching through textbooks, with students repeating well-established procedures demonstrated by the teacher. The criterion for performance assessment here is correct solutions identified in a "correct" way.

² Each level is characterized by teacher's conceptions of:

What mathematics is.

What it means to learn mathematics.

What teachers of mathematics teach students.

What teacher and student roles should be.

What constitutes evidence of student knowledge and criteria for judging correctness, accuracy, or acceptability of mathematical results and conclusions.

At level 1, teachers are convinced that mathematics is all about the rules but admit that the rules are backed up by concepts and procedures. There is an emerging awareness of the use of instructional representations in learning, and the teacher considers it necessary to make students understand the concepts and procedures and to teach them problem-solving skills.

Finally, teachers who have reached level 2 treat mathematics as a complex system of integrated concepts, procedures, and attitudes. They believe in teaching for understanding which is reached by engaging students in the process of actually “doing mathematics.” Such teachers underline the importance of making students work independently and creating conditions for them to express freely their opinion, while problem solving is understood as a method of teaching (teaching “through problem solving”, as compared to teaching “to solve problems’ characteristic of level 1) .

Pehkonen argues that level 3 and higher are also possible. The challenge is to bring teachers’ beliefs from lower levels to higher ones [Pehkonen, 1994. P. 195] .

Researchers identify different categories of beliefs: regarding mathematics as a science, or mathematics as a school subject, or the role of the teacher in teaching, or the role of students [Törner, 2002]. It has been put forward lately that teachers’ beliefs about the essence of mathematics, about teaching mathematics, and about teaching in general may be investigated in isolation [Liljedahl, Rösken, Rolka, 2007] .

One of the most important and productive classifications of teachers’ beliefs is one based on preferred approaches to teaching mathematics. Dionne and Ernest differentiated between traditional, formalist and constructivist (based on the view that knowledge is constructed by students) perspectives [Dionne, 1984; Ernest, 1991]. Thompson and her co-authors introduced the concept of “orientation in teaching mathematics’ and defined conceptually-oriented teachers who are primarily focused on a system of ideas, styles of thinking, and methods of their development, and calculationally-oriented teachers who give more attention to numbers, calculation procedures, and numerical results [Thompson et al., 1994]. Meanwhile, Askew and his co-authors distinguish between teachers with connectionist (building connections in mathematics and exploring various techniques of problem solving), transmissionist (direct transmission of knowledge) and discovery orientations toward teaching mathematics [Askew et al., 1997] .

Grigutsch, Raatz and Törner categorize beliefs by four aspects of mathematical belief system: aspect of scheme (mathematics as a rigid collection of rules and formulas), aspect of process (mathematics as a science consisting of problem solving processes), aspect of formalism (mathematics as a deductive and logical science), and aspect of application (mathematics as a science which is relevant for society and life) [Grigutsch, Raatz, Törner, 1998]. However, this classifica-



Table 1. **Description of approaches to mathematics teaching in accordance with the OECD model** [Brooks, Brooks, 1993. P. 17]

Traditional approach	Constructivist approach
The learning process is built around the basic skills	The learning process is built around the concept as a whole
Strict adherence to fixed curriculum is highly valued	Pursuit of student questioning is highly valued
Student is a "blank slate" to be filled with information provided by the teacher	Student is a thinker with emerging theories about the world
The teacher is normally didactic when transmitting knowledge to students	The teacher works interactively, mediating the environment for students for efficient learning
Teacher seeks the correct answer to validate student learning	The teacher is trying to understand the student's point of view in order to understand student's present conceptions for use in subsequent lessons
Students usually work individually	Students usually work in groups
Knowledge is assessed through tests, apart from the learning process	Assessment of knowledge is regarded as an integral part of the learning process and is performed through monitoring of students, their work and projects

tion is applicable to teachers' views about mathematics as a science rather than a school subject.

The trend most widely recognized by scientists is the differentiation between traditional and constructivist beliefs: the former are built around direct transmission of knowledge, while the latter suggest that students construct knowledge themselves through specifically organized activity [OECD, 2009]. This is the model we used in our research to evaluate teachers' beliefs and perceptions about good teaching (Table 1).

The term "constructivism" was first used in research of mathematics education in 1983, although it had been introduced in a more generalized context a bit earlier by Jean Piaget who used it in establishing genetic epistemology. The constructivist movement in mathematics education is believed to have been led by American researcher Ernst von Glasersfeld who defined the two fundamental principles of constructivism in 1975:

- 1) knowledge is not passively received but actively built up by the cognizing subject;
- 2) the function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality [Safuanov, 1999. P. 16].

Teachers with a constructive approach can be characterized by their perception of a student as an active participant in the process of gain-



ing knowledge. This kind of teacher gives students the opportunity to figure out solutions to problems by themselves. According to Kim Beswick, constructivism is the most effective medium to achieve the greatest results by students. [Beswick, 2007].

Traditionalist teachers believe that their main role is to present the material—clearly, precisely and structurally, to explain correct tasks solution and to maintain the necessary level of concentration in the classroom.

There are hardly any teachers who base their practices on only one of the conceptions described above; every teacher includes and integrates elements of both approaches in teaching. Nevertheless, Staub and Stern conducted a quasi-experimental study to find out that teaching with a more pronounced constructivist orientation was associated with better student performance than teaching with a more pronounced instructional orientation [Staub, Stern, 2002] .

2. International studies on mathematics teachers' beliefs

One of the earliest major research projects on teachers' beliefs was TALIS (OECD's Teaching and Learning International Survey). This international comparative survey examined a wide range of issues associated with the teaching profession and with teacher status in today's society, assessing teachers' job satisfaction and professional development, strategies and methods they apply in the classroom, school climate, teacher's beliefs, etc.

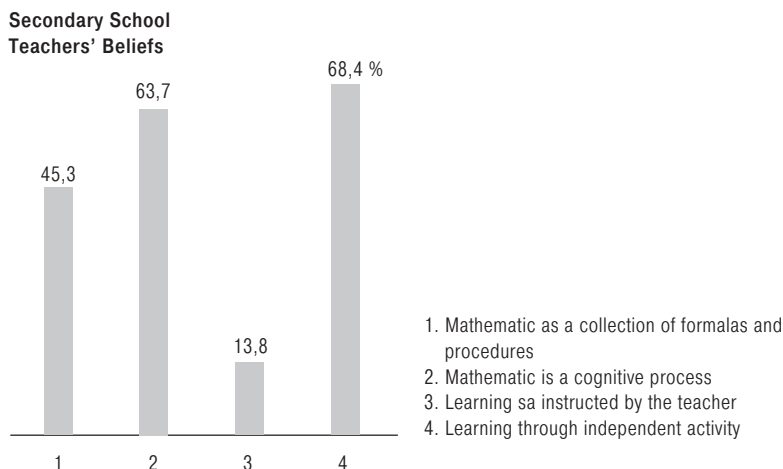
Over 2,000,000 teachers from 23 countries participated in the first wave of TALIS in 2008 [OECD, 2008]. The second wave was conducted in 2013–2014 and included the Russian Federation. TALIS has provided the basis and methodological procedure for developing other subject teacher research projects. In particular, the Nordic-Baltic Comparative Research in Mathematics Education (NoRBA), which served as a starting point for our own research and will be touched upon later, was largely based on the theoretical foundation and the questionnaire of TALIS.

TALIS is designed to collect information about teachers as a professional group and is not focused on teachers of mathematics. The latter were the specific focus of cross-cultural research TEDS-M studying teacher education systems and assessing quality of education provided for future teachers of mathematics in primary and secondary schools. The study was conducted in 2006 with Russia's participation.

The TEDS-M project covered about 22,000 future teachers of mathematics from 500 teacher education universities and analyzed about 750 mathematics teacher training programs in 17 countries. The sample was drawn from final-year students studying to become primary school teachers or teachers of mathematics in secondary school. The study also questioned teacher education professors [IEA, 2011] .



Figure 1. **Assessment of beliefs based on TEDS-M**
 (Russian sample)



The TEDS-M assessed professional knowledge of mathematics teachers, educational programs used in different countries, future teachers' beliefs, and also collected contextual information. Assessment of teachers' beliefs about mathematics and teaching mathematics measured two orientations, conceptual and calculational [Lester, 2007], which were close in their nature to the traditionalist and constructivist orientations described above [Kovalyova, Denishcheva, Sheveleva, 2011].

The survey of teachers didn't reveal any "pure" traditionalists or constructivists: teachers relied in their practice on a complex system of beliefs combining elements from different approaches. International experts believe Russia belongs to the group of countries where teachers have a rather constructivist orientation, most of them sharing the idea of "mathematics as a cognitive process" and that of "learning through independent activity" (Fig. 1).

A number of correlations have been revealed between the level of future teachers' professional achievements and beliefs. Thus, students sharing the constructivist ideas of "mathematics as a cognitive process" and "learning through independent activity" demonstrate a higher level of mathematical content knowledge and pedagogical knowledge as compared to those who "totally disagree" with such concepts [Kovalyova, Denishcheva, Sheveleva, 2011].

The TEDS-M target group was comprised of senior students of teacher education universities. Results of the project in Russia indicate that teacher education students who graduated in 2008 were going to develop their teaching practices based on the constructivist orientation. However, the survey proved that 73% of students did not perceive teaching as a promising career, 40% were sure they would

never actually work as teachers, and only 5% believed that teaching was their lifetime vocation. Besides, there is ample research confirming that teachers' beliefs may change after starting a teaching career [Murphy, Lee, Edwards, 2004] .

To assess characteristics of practicing teachers and conduct a comparative study of mathematics teachers' beliefs, we did our own survey using the NoRBA (Nordic-Baltic Comparative Research in Mathematics Education)³ questionnaire.

NoRBA is a comparative study of mathematics education in the Northern Baltic countries (Latvia, Lithuania, Finland, Sweden, and Norway). This study produced a questionnaire designed to explore secondary school teachers' beliefs regarding efficient mathematics teaching and learning [Lepik, Pipere, 2011]. The principal difference between this questionnaire and the one used in the TEDS-M is that the former is focused on teaching practices (investigates beliefs directly associated with teaching), while the latter is about studying beliefs regarding the nature of mathematics and the process of its teaching.

3. Method

3.1. Instrument

The NorBa project was launched relatively recently. In 2010, a group of researchers from Estonia, Latvia and Finland (Madis Lepik, Maraku Hannula, and Anita Pipere) developed a questionnaire to measure various aspects of mathematics teachers' beliefs in cross-culturally valid ways.

The basic part of the NorBa questionnaire includes five modules:

- 1) general information (social and demographic characteristics of teachers: age, years of experience, type of settlement where they are teaching, number of students in a class, etc.);
- 2) school climate (items assessing job satisfaction, relationship with colleagues and school administrators);
- 3) 3) general beliefs about teaching (two pools of items reflecting two learning approaches: constructivism and traditionalism);
- 4) conceptions of good teaching of mathematics;
- 5) teachers' perceptions of their own classroom practices (items on how often teachers use specific types of activities with students during the class).

Each module consists of a series of statements, for which respondents specify their level of agreement or disagreement measured using five- or four-point Likert scales. Therefore, the questionnaire reveals how teachers evaluate their own beliefs; however, the accu-

³ Official website of Nordic-Baltic Comparative Research in Mathematics Education (NoRBA) <http://norbal.wordpress.com/2011/09/29/hello-world/>.



mulated research on teachers' beliefs has led the academic community to agree that results of such questionnaires may be used as characteristics of beliefs.

The very first version of the questionnaire was developed in English and later translated into languages of the participating countries, including Russian (for Russian-speaking teachers of Latvia and Estonia). The survey was conducted in three countries, Latvia, Finland and Estonia, in 2010–2011 [Lepik, Pipere, 2011; Lepik, Pipere, Hannula, 2013; Lepik, 2005].

To provide successful research on the sample of Russian teachers, we used a Russian version of the NorBa questionnaire, which we modified with the developers' approval by paraphrasing some of the items to make them sound as smooth as possible in Russian, keeping the same meaning as the original items in English. The survey of Russian teachers was conducted in spring 2013.

As this paper is designed to explore mathematics teachers' beliefs, we are going to analyze the results using questionnaire modules 3, 4, and 5.

In Latvia, 390 teachers were surveyed, including 95 native Russian speakers (Russian-speaking teachers). The respondents were aged between 25 and 66 (average age was 46.7); average years of experience—23.3; the prevailing age range—from 40 to 49. The Latvian sample was selected to be representative of the general population of maths teachers [Sapkova, 2011].

In Estonia, 332 teachers from 15 regions were surveyed, including 92 native Russian speakers (Russian-speaking teachers). The respondents were aged between 25 and 77 (average age was 46.9); average years of experience—22.8⁴.

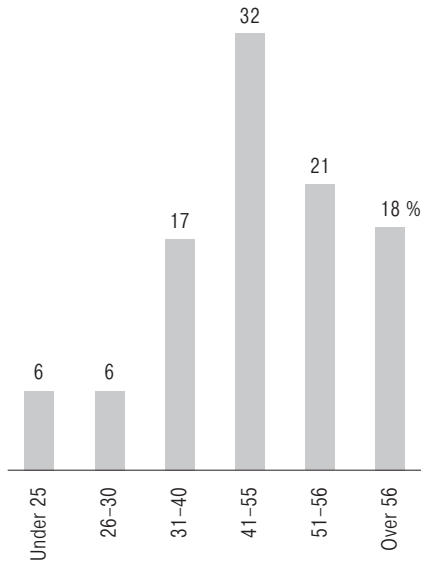
In Russian Federation, the sample consisted of 1,096 secondary school mathematics teachers in Krasnoyarsk Krai⁵, which accounted for 40% of the mathematics teachers' population in the region. A special analysis showed that the Russian sample may be considered representative of the regional mathematics teachers' population (the sample was compared to the population by type of settlement and type of educational institution, with differences between the sample and the population being under 3% for all the criteria).

The average age of Russian teachers was 46 years, while the average years of experience was 20 years. As shown in Figure 2, about

3.2. Sample and procedure

⁴ The NorBa questionnaire database for Latvia and Estonia was provided by questionnaire developers Madis Lepik and Markku Hannula based on the Terms of Collaboration For NorBa of September 13, 2013.

⁵ We express our thanks to the staff of Krasnoyarsk State Specialized Government Agency "Center for Education Quality Assessment" and personally to Lyubov Ryabinina, Deputy Head of the Center, and Yulia Koreshnikova, materials developer, for their assistance in conducting the survey.

Figure 2. **Teacher respondents' age** (Russia)

40% of teachers in the region are over 50 and only 12% are under 30. Senior people account for 18% of teachers.

Latvian and Estonian teachers were sent emails describing the survey and inviting them to participate. Those who agreed received a questionnaire, filled it out, and sent it back to the survey staff. Russian teachers followed the same procedure, except that they were provided access to an e-questionnaire to fill it out online. Teachers of all the three countries were reassured that all information collected by the survey would be kept in strict confidence and would be only available to the researchers.

3.3. Construction of scales

Item Response Theory [Kardanova, 2008] and Multiple Group Exploratory and Confirmatory Factor Analysis [Byrne, 2011] were used to construct scales, investigate their psychometric properties, and verify cross-cultural equivalence of the constructs measured. Detailed analysis of scale construction goes beyond this article and deserves a separate discussion. Nevertheless we will provide a brief description of the analysis, as reliable and valid scale scores are a prerequisite for international comparative studies. We will describe the process of scale construction using the example of module 3 of the NoRBA questionnaire, "General Beliefs about Teaching." Module 3 in the original version included sixteen items, of which twelve were placed on the constructivism scale and four on the traditionalism scale.

The data were analyzed using specialized software programs: Winsteps [Linacre, 2011] and Mplus Version 6.12 [Muthén, Muthén, 1998–2010] with the MLR estimator.



At the first stage of the research, exploratory factor analysis (with the Geomin rotation) was performed for the samples of each participant country in order to extract factorial invariance of a verifiable equivalence. All the three samples revealed a two-factor structure. Item 1 (*Problems that students encounter or will encounter in their everyday life play a significant role in development of their knowledge*) had low loadings on both factors, so it was omitted from the analysis, and the model was built on the remaining fifteen items (V2-V16).

Item Response Theory analysis with the Partial Credit Model showed similar results. In this case, principal component analysis of standardized residuals was used for a dimensionality study [Smith, 2002]. Item 1 was excluded again due to unsatisfactory psychometric characteristics.

The analysis showed that module 3 “General Beliefs about Teaching” of the NoRBA questionnaire consisted of two scales. These scales can be interpreted as traditionalism (grouping items 2, 3, 4 and 16) and constructivism (items 5–15). A factorial structure like that confirms the theoretical hypotheses of the questionnaire’s developers.

Afterwards the scales were analyzed separately. Each scale was found out to be unidimensional (i. e. serving to measure only one construct), all scale items had satisfactory psychometric characteristics and fit the measurement model well. Therefore, these scales can indeed be used to evaluate the level of teacher constructivism and traditionalism in each of the countries.

However, to compare the levels of teacher constructivism and traditionalism across countries, we should first demonstrate cross-cultural measurement invariance of the constructs. Differential Item Functioning (DIF) analysis was conducted for this purpose [Wang, 2008]. Data from a preliminary analysis suggested that some of the items were understood differently by teachers in different countries, which required additional research on the possibility of making comparisons.

We performed multiple group confirmatory factor analysis to verify cross-cultural equivalence of the measurement models in the three samples: Russian (N=343), Latvian (N=390), and Estonian (N=332). The Russian sample was originally larger than the other two; that is why we chose a random subsample which was used for further analysis.

Multiple group confirmatory factor analysis is usually comprised of the following steps [Byrne, 2011]: 1) measurement models are developed for each individual sample, 2) a set of common parameters is identified, and, finally, 3) constraints are set for equivalence of model parameter values. Constraints set on equivalence of factor loadings and intercepts of the observable variable indicate weak and strong invariance of factorial structures, respectively; meeting the requirement for strong equivalence allows to make a meaningful comparison of mean scores between the samples. Not meeting the requirements for the invariance of factor loadings or intercepts indicates non-uniform

Table 2. **Multiple group correlation coefficient**

Model	Model correlation coefficients				
	χ^2	<i>df</i>	<i>SCF</i>	<i>CFI</i>	<i>RMSEA</i> (90% <i>CI</i>)
M1. Configural invariance	421.89	254	1.073	0.917	0.043 (0.036–0.050)
M2. Factor loading invariance	478.17	280	1.082	0.901	0.045 (0.038–0.051)
M2*. Partial factor loading invariance	452.28	278	1.083	0.913	0.042 (0.035–0.049)
M3. Invariance of intercepts	979.17	304	1.079	0.664	0.079 (0.074–0.085)
M3*. Partial invariance of intercepts	473.42	293	1.078	0.910	0.042 (0.035–0.048)

or uniform bias, respectively [Matsumoto, Van de Vijver, 2012]. In practice, strong invariance as a prerequisite for comparison of mean scores between the samples is by no means always supported. In this case, the model can be constructed with partial measurement invariance to allow for a meaningful comparison of latent variable parameters (means, variances, covariances) between the samples.

For each sample, we developed individual baseline models that showed an acceptable fit with the data. At the next step, despite the fact that the baseline models demonstrated only partial structural equivalence, we verified invariance for the set of common parameters. To do this, we created a multiple group model M1 based on the combined sample (N=1,065) including both common and group-specific parameters. All factor loadings were set as free parameters, latent factor variances were fixed at 1 in each sample for identification, and means of latent factors were set to zero in the Russian sample. The resulting model demonstrated acceptable fit with the data (Table 2).

Table 2 uses the following conventions: χ^2 stands for the Satorra-Bentler scaled chi-square statistic, *df* stands for the number of degrees of freedom, *SCF* stands for the chi-square scaling correction factor, *CFI* stands for comparative fit index, *RMSEA* stands for the root mean square error of approximation with a 90% confidence interval. We used $CFI > 0.90$ and $RMSEA < 0.06$ values here and below as criteria of acceptable fit of the model to data [Byrne, 2011].

Factor loading invariance was tested at the next stage. Constraints for equivalence of loadings in the groups were added to model M2 (latent factor variances were set as free parameters). After that, constraints on equivalence were lifted one by one in the samples, based on analysis of modification indices, until the nested model (M2*) didn't differ significantly from the original one (M1) in terms of difference between their chi-square coefficients (with scaling corrections for the MLR estimator [Muthén, Muthén, 1998–2010]). Loadings of two items were found to be non-equivalent parameters, but



the corresponding chi-square values ($\chi^2=13$ in both cases) were relatively low, which suggests low non-uniform bias.

The uniform bias turned out to be much more pronounced. The model with constraints for invariance of intercepts (M3) demonstrated poor fit with the data. The most notable non-invariance was found for the intercepts of items 15 (*Assessment ought to be based on practical tasks, projects, and investigations*, $\chi^2=117$) and 6 (*Learners learn best by finding solutions to problems on their own*, $\chi^2=56$) in Russia, item 3 (*How much learners learn depends on how much background knowledge they have*, $\chi^2=55$) in Latvia, items 16 (*A quiet classroom is generally needed for efficient learning*, $\chi^2=102$) and 5 (*My role as a teacher is to facilitate learners' own inquiry*, $\chi^2=59$) in Estonia. Intercepts of some other items were also classified as non-invariant, with a much lower chi-square value, though. The fit of the constructed model M3* proved to be acceptable and was not significantly different from that of the model M2*. Parameters of latent factors in model M3* allowed for a substantive comparison between the samples. (Wald test was used to compare parameters.)

Thus, a number of questionnaire items are perceived differently in samples from different countries. The reasons for this may be numerous, from inaccurate translation to differences in educational systems of the participating countries. However, the number of such items is rather small, the rest of items proving equivalent. That being said, constructivism and traditionalism scales may be regarded as partially equivalent for the participating countries.

In order to assess the level of constructivism and traditionalism in participants' beliefs, we used the Partial Credit Model again, which enabled us to get estimates of parameters on the interval scale with precision characteristics specified. The interval scale made it possible to compare measurement results obtained for partially differing sets of items and hence to take into consideration partial non-equivalence of the scales. All non-equivalent items were treated as unique for their samples.

All estimates were converted to a 100-point scale with the mean value of 50 and standard deviation of 10 for convenience of comparison and interpretation.

Therefore, each teacher is characterized through two scores on a 100-point scale showing the level of constructivism and traditionalism in their beliefs.

As seen from the previous part, module 3 of the NoRBA questionnaire, "General Beliefs about Teaching and Learning," consists of two scales that can be interpreted as traditionalism (4 items) and constructivism (11 items). Both scales were acknowledged applicable to assess the level of constructivism and traditionalism in partic-

3.4. Assessment of participants

4. Conceptions of good teaching

Figure 3. **Means on the constructivism and traditionalism scales**

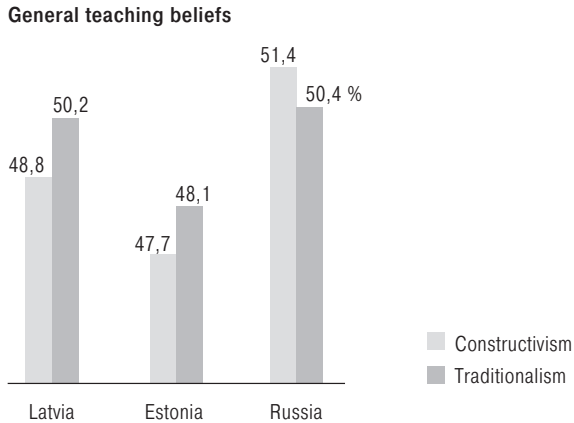


Table 3. **Descriptive statistics on the constructivism and traditionalism scales**

Country		N	Minimum	Maximum	Mean	Standard deviation
Latvia	Constructivism	390	9.5	85.6	48.8	9.2
	Traditionalism	390	18.3	86.3	50.2	10.6
Estonia	Constructivism	332	10.4	85.6	47.0	8.7
	Traditionalism	332	29.1	81.9	48.1	8.5
Russia	Constructivism	1,096	18.3	72.2	51.4	10.4
	Traditionalism	1,096	17.5	73.6	50.4	10.2

participants' beliefs. The reliability was 0.67 for the constructivism scale and 0.61 for the traditionalism scale (we used the Pearson Reliability index which Item Response Theory utilizes as an alternative to the classic reliability coefficient). Reliability indicators like this are satisfactory, given the small number of items for each of the scales. Table 3 provides descriptive statistics on the scales.

Figure 3 shows the mean level of constructivism and traditionalism in teachers' beliefs across the countries on a 100-point scale.

Statistical evaluation of the differences between countries on these scales was performed through single factor analysis of variance (ANOVA). We found that teachers in different countries varied significantly in their level of both constructivism ($F(2; 1,808) = 27.97, p < 0.001$) and traditionalism ($F(2; 1,808) = 6.87, p < 0.001$).

However, we made multiple comparisons by means of the Least Significance Difference (LSD) method to find out that pairwise com-



Table 4. **Correlation between the constructivism and traditionalism scales**

Country	Correlation
Latvia	0,02
Estonia	-0,18**
Russia	-0,05

** $p < 0,01$.

parison revealed no differences on the traditionalism scale between Russia and Latvia, which means that the mean level of traditionalism is the same for Russian and Latvian mathematics teachers.

Thus, Russian teachers of mathematics are much more constructivist than teachers in the other two countries. Estonian teachers have less traditionalist beliefs than teachers in Russia and Latvia, but the level of constructivism is also lower in Estonia than in the other two countries.

Teachers' beliefs have nothing to do with age: correlation analysis revealed no statistically significant correlations either in the combined sample or in each of the samples. Constructivist beliefs about teaching are positively associated with school climate (the correlation is statistically important, $r = 0.23$, $p < 0.01$). Traditionalist beliefs are affected by neither school climate nor teachers' age. School climate was assessed using one of the questionnaire modules. The module includes nine items related to job satisfaction and relationship with colleagues and school administrators, which served as the basis for the scale.

Correlations between the constructivism and traditionalism scales are non-significant for all the countries except Estonia, where the correlation is negative and rather weak (Table 4) .

Zero correlations between the scales demonstrate that any teacher may be both constructivist and traditionalist at the same time, which is why it's impossible to classify them rigidly into one of two categories.

For this reason, we suggested that there should be profiles of beliefs that would combine constructivist and traditionalist features displayed with different intensity. To draft such profiles, we conducted a hierarchical cluster analysis (Ward's method), having chosen the levels of traditionalism and constructivism as clustering factors. Dispersion of distances within each cluster is minimized with each stage of clustering in this method. Milligan argues that this method is the optimal choice in lots of cases [Milligan, 1996] .

To get a stable clustering solution, the sample was cut by 12 teachers whose scale values selected for clustering differed from the mean value by more than three standard deviations, i. e. diverged

Table 5. **Mean values across the clusters**

Scale	Cluster No.								
	1	2	3	4	5	6	7	8	9
N (number of observations)	230	159	150	116	269	323	272	217	60
Constructivism	57	53	58	65	51	41	39	47	73
Traditionalism	56	66	37	44	48	58	46	38	56
Constructivism level	high	medium	high	high	medium	medium	low	medium	high
Traditionalism level	high	high	low	medium	medium	high	medium	low	high

 Table 6. **Threshold values across the scales**

	Low	Medium	High
Constructivism	points ≤ 39	39 < points ≤ 53	53 < points
Traditionalism	points ≤ 40	40 < points ≤ 52	52 < points

substantially from the general trend. We tested various versions of clustering solutions based on interpretability criteria and on factors of internal/external validity. Internal validity was assessed through analysis of statistical differences between clusters on the constructivism and traditionalism scales with the help of analysis of variance (ANOVA): a clustering solution is only valid if differences between the clusters are significant. As for external validity, it was also analyzed by assessing differences between the clusters, but with the use of an “external” variable that did not belong to clustering factors. We assessed external validity using the results obtained on the scales of questionnaire module 4 designed to evaluate beliefs about efficient teaching of mathematics. Assessment of external validity will be described in the next part of this paper.

Hierarchical clustering provided nine clusters. The number was chosen because the questionnaire authors [Lepik, Pipere, Hannula, 2013] used a similar classification. They had suggested that the constructivism and traditionalism scales could each be divided into three levels: high, medium, and low. The researchers believe a solution like that is the best to describe the total sample, allowing for a logical interpretation and being valid both internally and externally. Table 5 provides mean constructivism and traditionalism values for the resulting clusters on a 100-point scale.

For the purpose of further interpretation of the drafted profiles, we set thresholds allowing for differentiation of teachers by their lev-



Table 7. **Profiles of mathematics teachers' beliefs about efficient teaching**

		Traditionalism		
		Low	Medium	High
Constructivism	Low		Anti-constructivist	
			Latvia: 14.2%	
			Estonia: 22.0%	
			Russia: 13.4%	
			Total: 15.1%	
	Medium	Anti-traditionalist	Modest Compromise	Traditionalist
		Latvia: 14.7%	Latvia: 15.7%	Latvia: 29.4%
		Estonia: 17.1%	Estonia: 20.5%	Estonia: 22.3%
		Russia: 9.7%	Russia: 13.1%	Russia: 27.3%
		Total: 12.1%	Total: 15.0%	Total: 26.9%
	High	Radical constructivist	Constructivist	Reconciliation of polarities
		Latvia: 10.5%	Latvia: 4.5%	Latvia: 11.0%
		Estonia: 7.6%	Estonia: 2.8%	Estonia: 7.6%
		Russia: 7.8%	Russia: 8.3%	Russia: 20.4%
		Total: 8.4%	Total: 6.5%	Total: 12.8%

els of constructivism and traditionalism. Threshold values were determined based on the mean initial points earned by respondents in the scale items. Most teachers selected the “Agree” or “Totally agree” point when responding to the items in this module. On this basis, we divided the scales as follows: the mean initial points of 3.5 or lower accounted for the low level on the constructivism scale; the MIP between 3.5 and 4.5 accounted for the medium level; and the MIP over 4.5 accounted for the high level. The traditionalism scale levels were determined similarly, but the MIP scale threshold values were set at 2.5 and 3.5, as the distribution of teachers on the traditionalism scale was less shifted towards positive response. Finally, we converted the thresholds to a logit scale and to a 100-point scale. The 100-point scale thresholds are given in Table 6.

We used these threshold values to classify teachers depending on their conceptions of good teaching. With respect to the threshold values, we grouped together clusters 1 and 9, 2 and 6. Table 7 shows distribution of teachers across the resulting profiles.

Most Russian teachers either prefer constructivism over traditionalism or follow the “reconciliation of polarities’ profile, which combines both approaches utilized to their maximum extent. Tradition-

alist teachers, however, are also numerous in Russia, accounting for 27% of the sample, which is close to Latvian 29%.

Let's discuss the most prominent of the resulting profiles.

Modest Compromise

This profile of beliefs is mostly typical of Estonian teachers. Such teachers share both constructivist and traditionalist conceptions of good teaching. They do not make students memorize rules and rigid solution procedures, but they are not enthusiastic about open learning, discussions with students, or getting them to work in small groups either.

Radical constructivist

This group includes 8% of all the teachers. Teachers of this profile are most numerous in Latvia—10.5% (as compared to 7.8% in Russia and 7.6% in Estonia). They see teaching as a joint participation with students in the effort towards knowledge construction. Such teachers prefer teaching small groups, helping students do their own research and making discoveries, relating course material to real-life problems. Their main goal is to facilitate conceptual understanding of mathematics without placing special focus on teaching formal skills.

Reconciliation of polarities

Teachers who base their practices on both approaches account for 13% of the total sample, being most numerous in Russia—20.4% (as compared to 7.6% in Estonia and 11.0% in Latvia). Such teachers likely organize their teaching to develop conceptual understanding of mathematics among students, at the same time giving a lot of attention to instrumental view of mathematics and focusing on rules and procedures.

5. Teachers' beliefs about good teaching/learning of mathematics

In the original version of the questionnaire, module 4 "Teachers' beliefs about Good Teaching/Learning of Mathematics" included 26 items designed to evaluate mathematics teaching beliefs. The conception actively used today divides such beliefs into three groups: "toolbox aspect", "process aspect", and "system aspect" [Lepik, Pipere, 2011].

In the "toolbox aspect", mathematics is seen as a set of rules, formulas, skills and procedures. According to this perception, mathematics learning is understood as using rules and formulas, mastering procedural skills. This perception is close to traditionalist teaching beliefs. The "system aspect" stresses rigorous proof, logic, exact definitions and a precise use of the mathematical language; mathematics is understood as a system. In the "process aspect", mathematics is considered as a constructive process in which relations among different notions play an important role. This perception sees learning as a process of knowledge construction with the paramount focus on de-

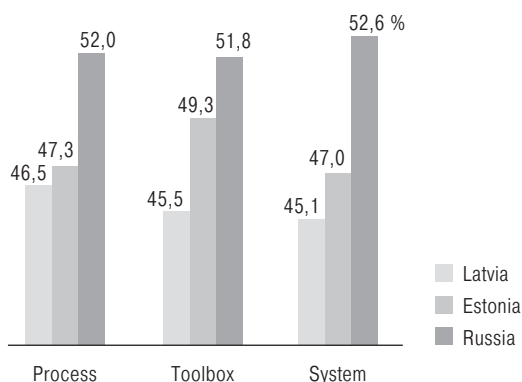


Table 8. **Descriptive statistics across the scales**

Country		N	Minimum	Maximum	Mean	Standard deviation
Latvia	Process aspect	390	10,5	81,2	46,5	8,7
	Toolbox aspect	390	15,8	70,3	45,5	9,1
	System aspect	390	23,2	77,9	45,1	8,6
Estonia	Process aspect	332	29,2	70,2	47,3	7,9
	Toolbox aspect	332	24,6	90,3	49,3	9,4
	System aspect	332	27,7	77,9	47,0	8,3
Russia	Process aspect	1095	24,0	81,2	52,0	10,5
	Toolbox aspect	1095	20,7	90,3	51,8	9,9
	System aspect	1095	10,7	77,9	52,6	10,0

Figure 4. **Means for Toolbox, Process and System scales**

Mathematic teaching beliefs



velopment of thinking processes and creative steps in mathematical activity. This perception is close to constructivism.

To evaluate teachers' beliefs about the most efficient approach to teaching mathematics, we developed three scales using methods similar to those described above in respect to the constructivism and traditionalism scales. Thus, module 4 is represented by the following scales:

- 1) "process aspect": 10 items, reliability: 0.8;
- 2) "toolbox aspect": 5 items, reliability: 0.65;
- 3) "system aspect": 6 items, reliability: 0.72.

Table 8 provides descriptive statistics for the scales in this part of the questionnaire.

Figure 4 is a graphical representation of mean values of the scales.

Table 9. **Correlation analysis of beliefs**

		Constructivism	Traditionalism
Latvia	Process aspect	0,63**	-0,17**
	Toolbox aspect	-0,19**	0,55**
	System aspect	0,13**	0,18**
Estonia	Process aspect	0,54**	0,09
	Toolbox aspect	0,16**	0,50**
	System aspect	0,29**	0,34**
Russia	Process aspect	0,57**	0,05
	Toolbox aspect	0,05	0,48**
	System aspect	0,26**	0,27**
Total sample	Process aspect	0,59**	0,03
	Toolbox aspect	0,05	0,49**
	System aspect	0,27**	0,26**

** $p < 0,01$.

Statistical evaluation of differences between teachers from different countries for the three scales was performed using single-factor analysis of variance (ANOVA). As a result, we revealed significant differences on all of the scales: Process ($F(2; 1,813) = 62.39, p < 0.001$); Toolbox ($F(2; 1,812) = 61.56, p < 0.001$), System ($F(2; 1,812) = 112.153, p < 0.001$).

Russian teachers demonstrated the highest, and Latvian the lowest mean values on all of the scales. Pairwise comparison showed that Estonian and Latvian teachers didn't have any significant differences on the Process scale. "Process aspect" teaching beliefs are much less widespread in the Baltic countries than in Russia, which confirms the results we have obtained earlier on general teaching approaches.

The System scale revealed the most substantial differences between Russian and Baltic teachers, which confirms the notion that Russia still enjoys traditions of high quality mathematics education with the focus on proof and on appropriate use of the mathematical language.

Theoretically, the "process aspect" in mathematics teaching beliefs correlates with the constructivist paradigm, and the "toolbox aspect" with the traditionalist one. This hypothesis was verified through a correlation analysis (Table 9).

We can see a significantly strong correlation between the constructivism scales and the "process aspect", the traditionalism scales and the "toolbox aspect" for all the three countries. These results were to be expected, because the scales had a very similar nature.



The System scale coefficient correlates with both constructivism and traditionalism for all the teachers. This curious result means that teachers of all countries regard proving processes and use of precise mathematical language as important components of mathematical training, no matter what their general teaching approach is.

In order to make theoretical and statistical connections between the traditionalism and Toolbox scales, as well as the constructivism and Process scales, first we need to be assured of the external validity of the clustering solution we had come to after analyzing the constructivism and traditionalism scales. The Process and Toolbox scales were not clustering factors as such, but they were closely related to the constructivism and traditionalism scales. To judge the clustering solution as valid, we needed to verify the significance of differences between the clusters for these scales. If the differences were significant, a cluster solution would be judged valid.

Analysis of variance (ANOVA) revealed that clusters differed significantly on the Process scale ($F(8; 1,785) = 91.24, p < 0.001$) and on the Toolbox scale ($F(8; 1,784) = 56.78, p < 0.001$), which proved the validity of the clustering solution.

Aiming to assess the relationship between teachers' beliefs and practices, we used module 5 of the NorBa questionnaire, "Classroom practices", which included six items divided into three categories depending on their content and pair correlations identified during the analysis of the whole sample of three countries (Table 10).

Category A may be classified as traditionalist, categories B and C as constructivist. Hence, we may suggest that the more constructivist the teacher, the more they assign activities of categories B and C to their students; contrariwise, the more traditionalist the teacher, the more often they give students category A activities.

We introduced three indices for each problem category. Indices were calculated by summing up points earned in two items and dividing the sum by 8 (maximum possible points). Indices may be ranged from 0.25 to 1. Next, we calculated Pearson correlation between the

6. Relationship between teachers' beliefs and practices

Table 10. **Distribution of module 5 items by categories**

Category A	Traditionalism	G1. Memorize formulas and rules G2. Solve standard problems using facts, notions and rules
Category B	Constructivism	G3. Deal with problems that have no obvious solutions G5. Develop your own algorithm to solve challenging problems
Category C	Constructivism	G4. Relate material learned in mathematics classes to everyday life G7. Work as researchers: try to find regular patterns, formulate statements and prove them

Table 11. **Relationship between teachers' beliefs and practices**

		Category A	Category B	Category C
Constructivism	Latvia	-0,08	0,12*	0,19**
	Estonia	0,01	0,19**	0,30**
	Russia	-0,04	0,11**	0,20**
Traditionalism	Latvia	0,32**	-0,01	-0,13**
	Estonia	0,23**	0,06	0,10
	Russia	0,20**	0,01	-0,02

** $p < 0,01$; * $p < 0,05$.

constructivism and traditionalism scales and the resulting indices for each of the participating countries. The results are presented in Table 11.

Our hypothesis was confirmed. Indeed, teachers transfer their conceptions of good teaching to their practices. In all countries, constructivist teachers were shown to use mostly problems from categories B and C. In Latvia, there is also an inverse relationship between traditionalism and category C: the more traditionalist the teacher's orientation, the less often they ask students to relate material to everyday life or give them research tasks. Use of category A problems correlates positively with the traditionalist approach in all of the three countries.

7. Comparison of Russian-speaking teachers

About 25% of school students in Latvia and 19% in Estonia go to Russian-language schools. We compared mathematics teaching beliefs of Russian teachers to those of Latvian and Estonian teachers working at Russian-language schools, native speakers of the Russian language. The Latvian subsample of Russian-speaking teachers consisted of 95 people (25% of the sample), while the Latvian subsample included 92 respondents (28% of the sample).

Figure 5 shows average results on the constructivism and traditionalism scales, and Figure 6 on the scales of module "Mathematics Teaching Beliefs".

Analysis of variance (ANOVA) revealed that differences between the countries were statistically significant for the constructivism scale and non-significant for the traditionalism scale. Differences between mathematics teaching beliefs in different countries are statistically significant for all the three scales (Table 12). Russian teachers demonstrated a significantly higher level of constructivism compared to Russian-speaking teachers in Estonia and Latvia. Russian teachers also have a constructivist orientation towards teaching specifically mathematics, their mean values on the Process scale being



Figure 5. Mean values on the scales of module 3 of the questionnaire for the subsample of Russian-speaking teachers

General teaching beliefs

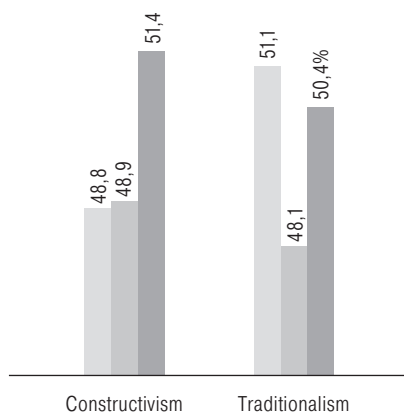


Figure 6. Mean values on the scales of module 4 of the questionnaire for the subsample of Russian-speaking teachers

Mathematic teaching beliefs

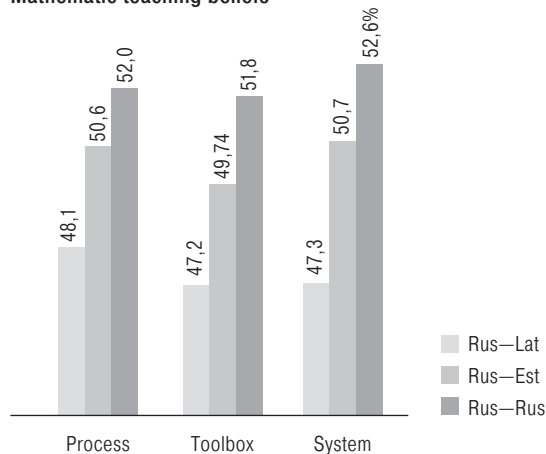


Table 12. Results of ANOVA analysis of beliefs for the subsample of Russian-speaking teachers

		<i>df</i>	<i>F</i>	Value
Constructivism	Between the groups	2	4,54	0,011
	Within the groups	1279		
	Total	1281		
Traditionalism	Between the groups	2	2,49	0,084
	Within the groups	1279		
	Total	1281		
Process	Between the groups	2	6,99	0,001
	Within the groups	1278		
	Total	1280		
Toolbox	Between the groups	2	10,73	0,000
	Within the groups	1277		
	Total	1279		
System	Between the groups	2	13,95	0,000
	Within the groups	1277		
	Total	1279		

Table 13. **Significance of differences in beliefs among Russian-speaking teachers in different countries**

	Rus vs. Lat_Rus	Rus vs. Est_Rus	All countries
Constructivism	Differences	Differences	Differences
Traditionalism	No difference	Differences	No difference
Process	Differences	No difference	Differences
Toolbox	Differences	Differences	Differences
System	Differences	No difference	Differences

much higher than those of their Russian-speaking colleagues from Latvia and Estonia.

Table 13 provides results of pairwise comparison of beliefs held by Russian teachers vs. beliefs held by Russian-speaking teachers in Latvia and Estonia.

The constructivism, traditionalism and Toolbox scales demonstrate significant differences between Russian and Estonian Russian-speaking teachers, while differences between Russian and Latvian Russian-speaking teachers can be found on every scale except the traditionalism one.

8. Conclusion

The TIMSS and PISA projects documented differences in levels of mathematical performance between Russian, Latvian and Estonian school students. With the highest TIMSS scores, Russian students showed considerably lower PISA performance than their Latvian and especially Estonian counterparts. We can thus presuppose that teachers' beliefs about teaching displayed through their classroom practices differ across these countries, too.

Mathematic education systems in Russia, Latvia and Estonia had a half century of common history but went separate ways in 1991. The Baltic countries joined the European Union and have been mostly building their education by learning from western models. It would thus be quite natural to suggest that today mathematics teachers' beliefs about teaching in general and efficient teaching of mathematics in particular differ from those of their Russian colleagues.

Our research revealed that differences between teachers of mathematics in Russia, Latvia and Estonia were statistically significant on all the scales analyzed. Russian teachers proved to be much more constructivist both in their general teaching approaches and in their conceptions of good teaching of mathematics: a high level of constructivism is typical for 36% of teachers in Russia, as compared to 26% in Latvia and 18% in Estonia. That is to say, Russian teachers try to focus on the overall conception more, to respond to students' de-



mands, and to pay special attention to interactive work when teaching. They see teaching mathematics as a constructive process organized around development of thinking skills. At the same time, a considerable percentage of Russian teachers, namely 27%, are traditionalist, which proves that teaching mathematics as a collection of rules, formulas and procedures is still rather popular in Russia.

20% of Russian teachers, 8% of Estonian teachers and 11% of Latvian teachers use both approaches at the same time in their practices. We can suggest that they teach to develop conceptual understanding of mathematics among learners, at the same time considering the instrumental view of mathematics and focusing on rules and procedures, provided that their students are heterogeneous in their level of training.

Most Estonian teachers share both traditionalist and constructivist beliefs about teaching, both moderately explicit. They reach a kind of balance between the two teaching approaches: Estonian teachers' conceptions of good teaching combine understanding of teaching as knowledge construction and as knowledge transfer. Conventional teaching focused on procedures and modern constructivist teaching methods developing conceptual understanding of material are seen as complementary rather than opposed.

The proportion of teachers with low levels of traditionalist attitude is higher in Latvia and Estonia (about 25% in both countries) than in Russia (17.5%). Apparently, Baltic teachers integrated in the European community are trying to get rid of obsolete and routine teaching methods.

General teaching beliefs correlate with teachers' conceptions of good teaching of mathematics. Constructivist teachers tend to see mathematics as a process; traditionalist teachers believe that teaching mathematics as a set of tools is best. However, mathematics teachers in all countries, whether they have a constructivist or traditionalist orientation, regard system as an important teaching factor and the use of proof and precise mathematical language as an integral part of mathematics education. Russian teachers have the highest average score on the system scale, which proves that Russia maintains a powerful tradition of high quality mathematics education. Focus on rigorous proof, logic, precise definitions and skillful use of the mathematical language is a characteristic feature of mathematics education in Russia.

Teachers in all participating countries have beliefs congruent with their practices.

In addition, we compared beliefs held by Russian teachers to those of their Russian-speaking colleagues from Latvia and Estonia. Expectedly, Russian-speaking teachers in the Baltic nations are closer in their beliefs to their Russian counterparts than teachers teaching in titular nation languages. Nevertheless, statistically significant differences were discovered between Russian teachers and

Russian-speaking Baltic teachers on the same scales as when comparing the national samples.

Russian teachers and Russian-speaking teachers in Estonia share the conception of good teaching of mathematics as a process and system (they have similar points on the Process and System scales). Differences between Russian teachers and Russian-speaking Latvian teachers are significant on all of the scales, except for traditionalism.

Thus, the research has shown that different approaches to education reform used in Russia and in the Baltic countries resulted in significant differences in mathematics teachers' beliefs about teaching mathematics. In particular, the Baltics have more teachers with low levels of traditionalist orientation than in Russia, which to some extent explains better PISA performance of Estonian and Latvian students.

Analysis of cross-cultural differences in teachers' beliefs provides essential information about teachers' classroom practices and choice of teaching strategies. This data allows to evaluate the situation in general school education more accurately and to predict its further development, which is especially relevant in light of education reforms.

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